Kolmogorov Turbulence, completed; then Geometrical Optics for AO

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Finish up discussion of Kolmogorov Turbulence from previous lecture

Structure function for atmospheric fluctuations, Kolmogorov turbulence

- Scaling law: $v^2 \sim \varepsilon^{2/3} l^{2/3} \sim r^{2/3}$ where r is spatial separation between two points

- Heuristic derivation: Velocity structure function ~ v^2

$$D_v(r) \equiv \left\langle \left[v(x) - v(x+r) \right]^2 \right\rangle \propto r^{2/3} \quad \text{or} \quad D_v(r) = C_v^2 r^{2/3}$$

- Here C_v^2 = a constant to clean up "look" of the equation. Describes the strength of the turbulence.

- For example:
  C_v^2 might be a function of altitude h: $C_v^2(h)$

What about temperature and index of refraction fluctuations?

- Temperature fluctuations are carried around passively by velocity field (incompressible fluids).

- So *T* and *N* have structure functions similar to *v*:

$$D_T (r) = < [T (x) - T (x + r)]^2 > = C_T^2 \; r^{2/3}$$

$$D_N (r) = < [N (x) - N (x + r)]^2 > = C_N^2 \; r^{2/3}$$

How do you measure index of refraction fluctuations in situ?

- Refractivity $N = (n - 1) \times 10^6 = 77.6 \times (P / T)$

- Index fluctuations $\delta N = -77.6 \times (P / T^2) \delta T$

$$C_N = \left(\partial N / \partial T \right) C_T = -77.6 \times \left(P / T^2 \right) C_T$$

$$C_N^2 = \left(77.6 P / T^2 \right)^2 C_T^2$$

- So measure δT , p, and T; calculate C_N^2

Simplest way to measure C_N^2 is to use fast-response thermometers

$$D_T (r) = < [T (x) - v (T + r)]^2 > = C_T^2 \; r^{2/3}$$

- Example: mount fast-response temperature probes at different locations along a bar:

X X X X X X

- Form spatial correlations of each time-series *T(t)*

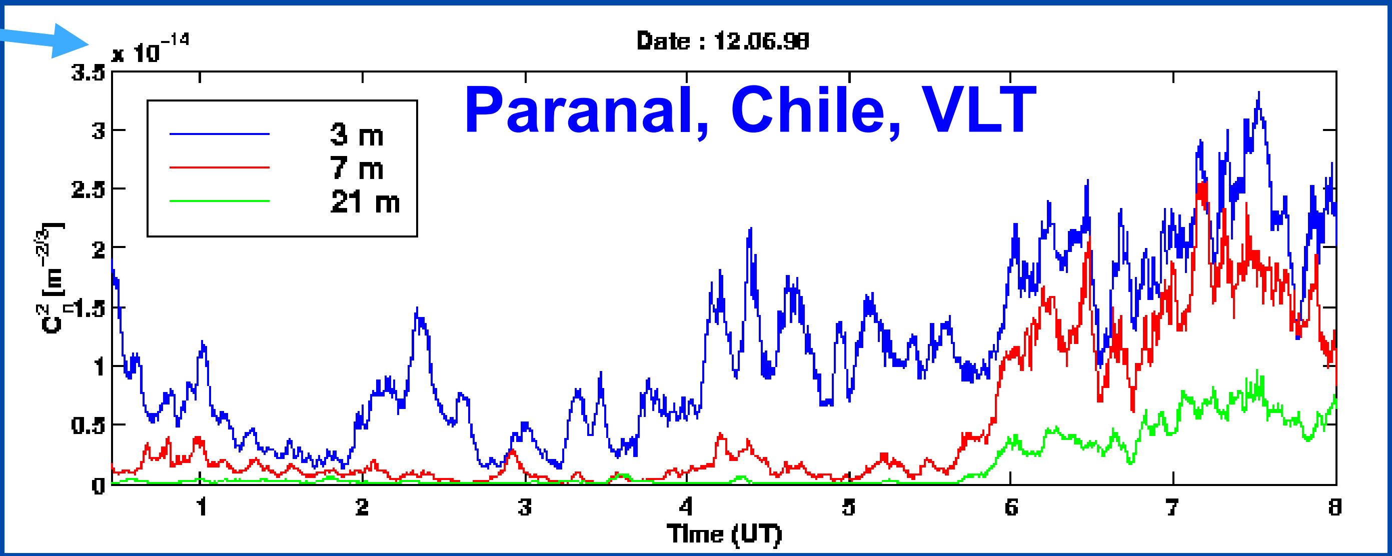
Typical values of C_N^2

- Index of refraction structure function

$$D_N(r) = \langle [N(x) - N(x+r)]^2 \rangle = C_N^2 \, r^{2/3}$$

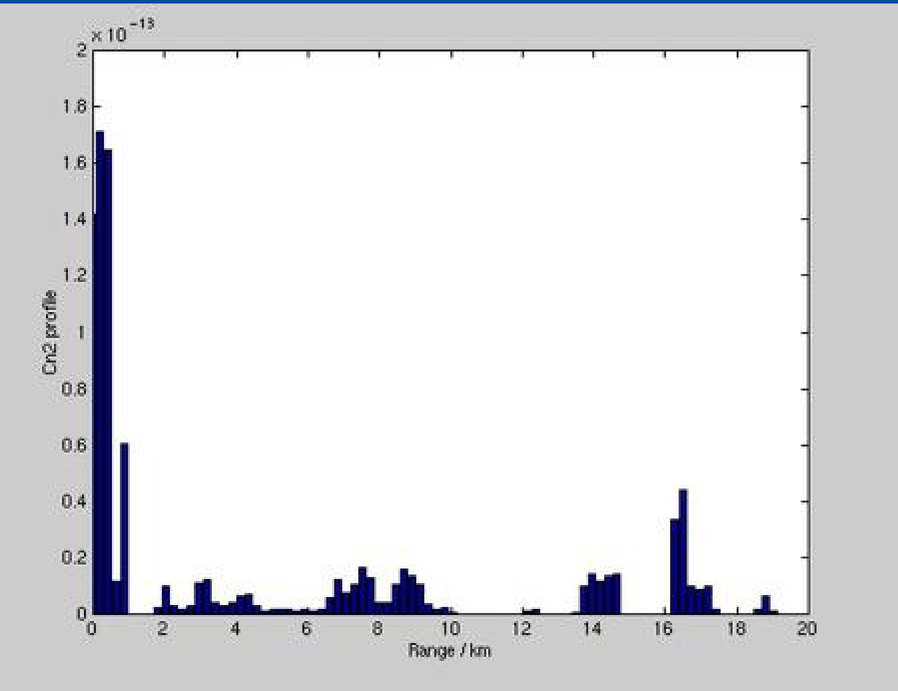
- Night-time boundary layer: $C_N^2 \sim 10^{-13} - 10^{-15}\ m^{-2/3}$

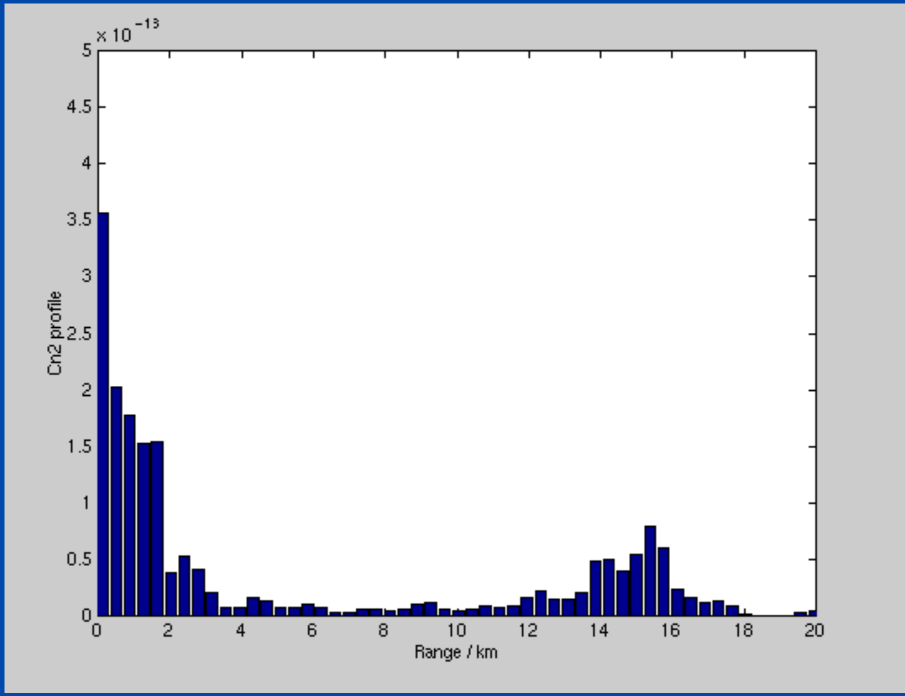
10^{-14}



Turbulence profiles from SCIDAR

Eight minute time period (C. Dainty, NUI)





Siding Spring, Australia

Starfire Optical Range, Albuquerque NM

Atmospheric Turbulence: Main Points

- Dominant locations for index of refraction fluctuations: atmospheric boundary layer and tropopause
- Atmospheric turbulence (mostly) obeys Kolmogorov statistics
- Kolmogorov turbulence is derived from dimensional analysis (heat flux in = heat flux in turbulence)
- Structure functions derived from Kolmogorov turbulence:

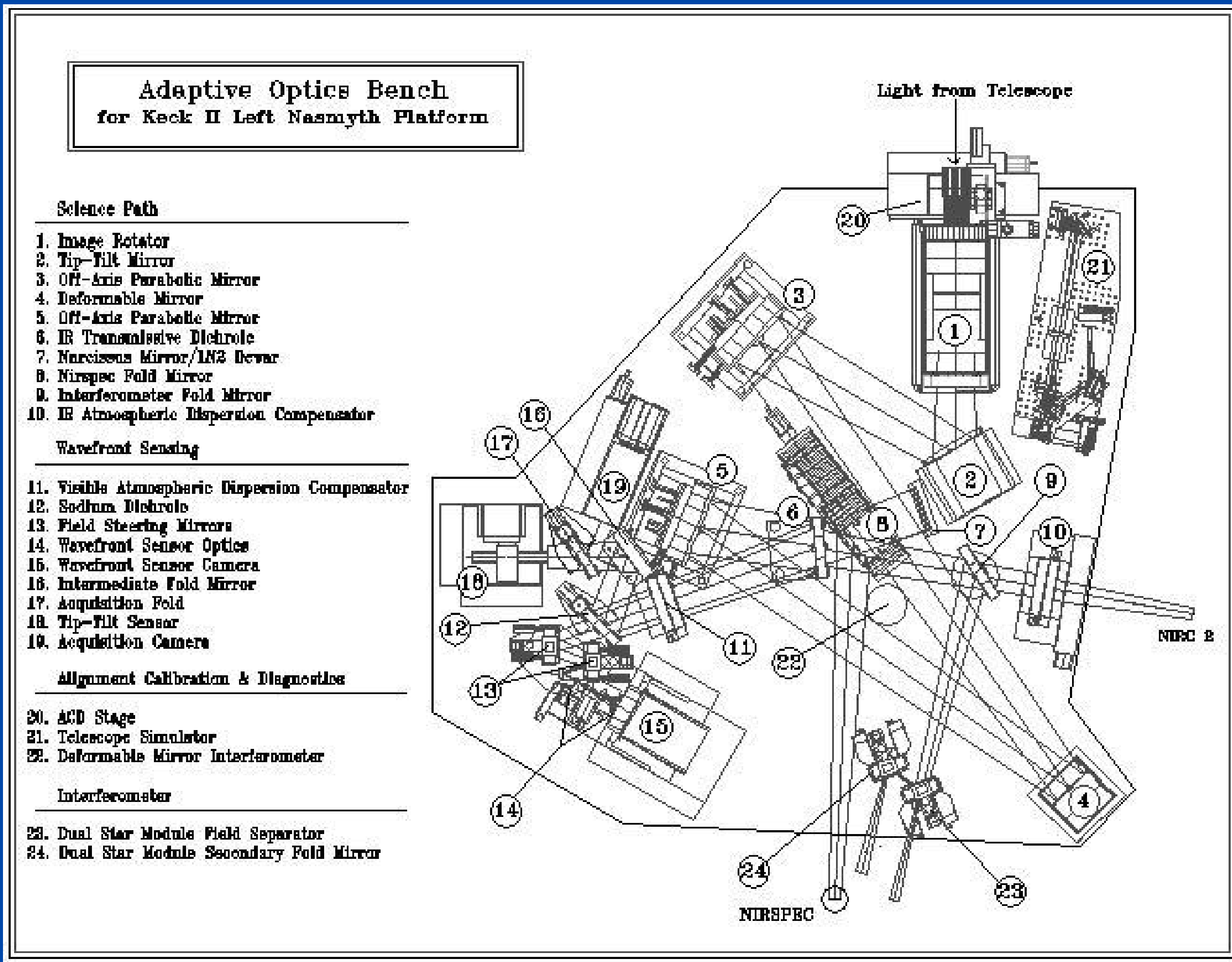
$$D_N(r) \equiv \left\langle \left[N(x) - N(x+r) \right]^2 \right\rangle \propto r^{2/3} \quad \text{or} \quad D_N(r) = C_N^2 r^{2/3}$$

- All else will follow from these points!

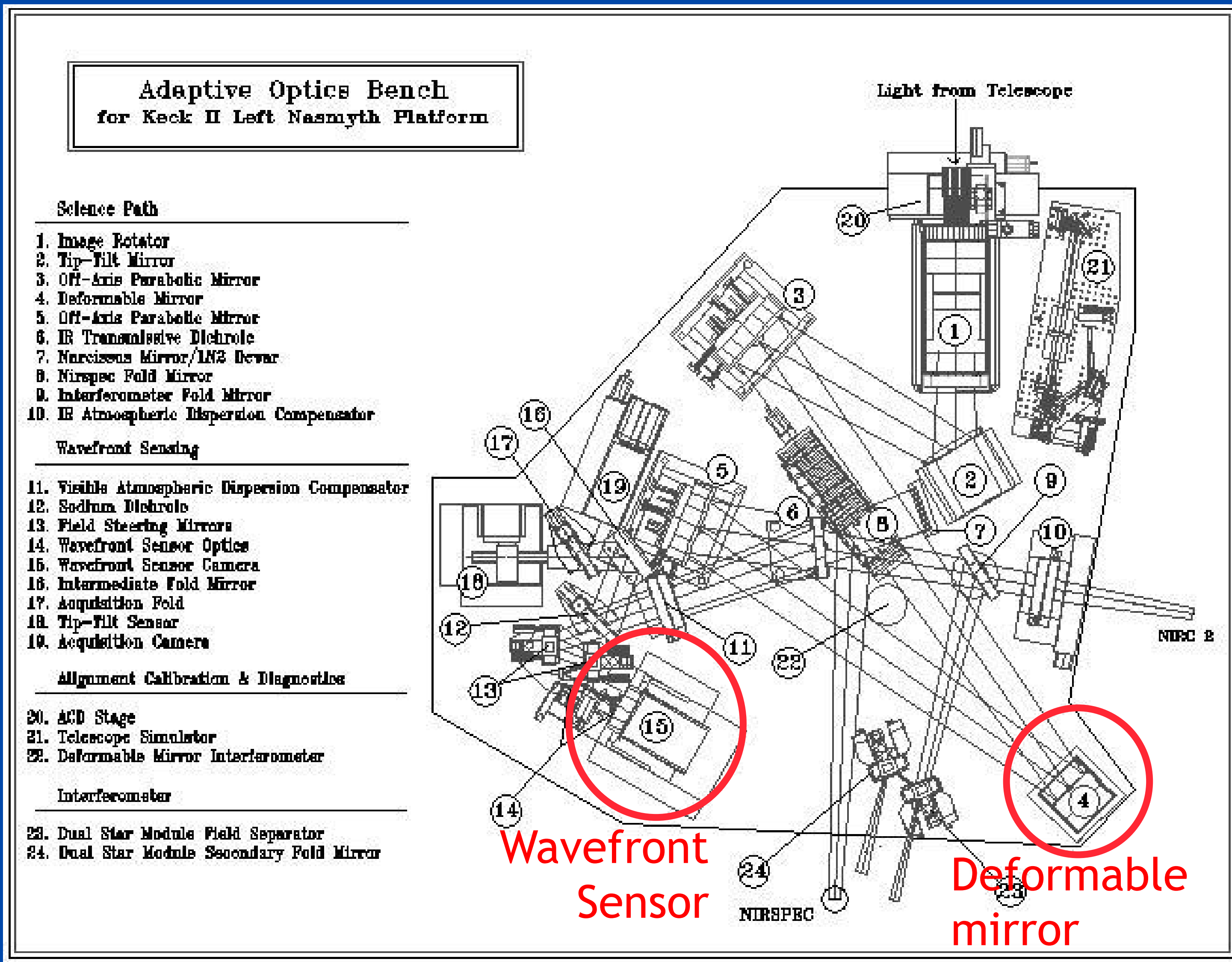
Goals: Geometrical Optics Review

- Basics of Geometrical Optics
  - Understand the tools used for optical design of AO systems
  - Understand what wavefront aberrations look like, and how to describe them
  - Characterization of the aberrations caused by turbulence in the Earth's atmosphere
- Application to the layout of an AO system

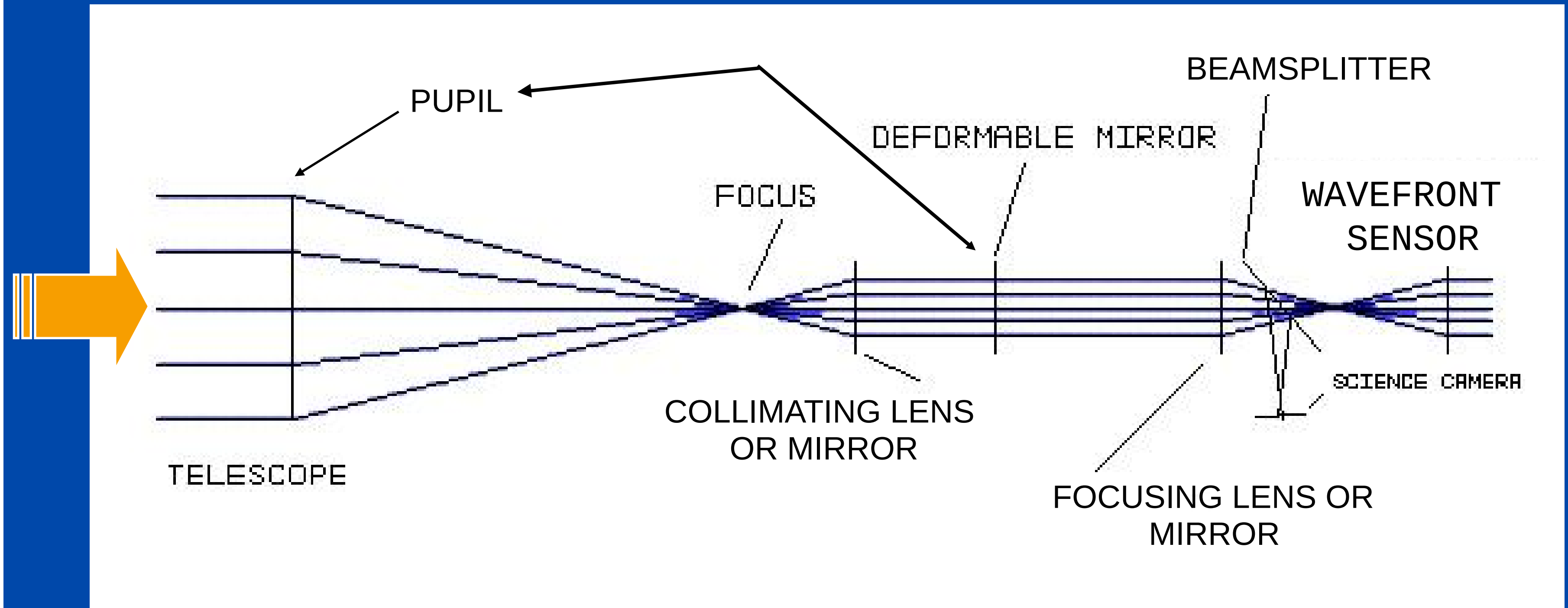
Keck AO system optical layout: Why on earth does it look like this ??



Keck AO system optical layout:
Why on earth does it look like this ??



Simplest schematic of an AO system



Optical elements are portrayed as transmitting, for simplicity: they may be lenses or mirrors

What optics concepts are needed for AO?

- Design of AO system itself:
  - What determines the size and position of the deformable mirror? Of the wavefront sensor?
  - What does it mean to say that "the deformable mirror is conjugate to the telescope pupil"?
  - How do you fit an AO system onto a modest-sized optical bench, if it's supposed to correct an 8-10m primary mirror?

- What are optical aberrations? How are aberrations induced by atmosphere related to those seen in lab?

Levels of models in optics

Geometric optics - rays, reflection, refraction

⇓

Physical optics (Fourier optics) - diffraction, scalar waves

⇓

Electromagnetics - vector waves, polarization

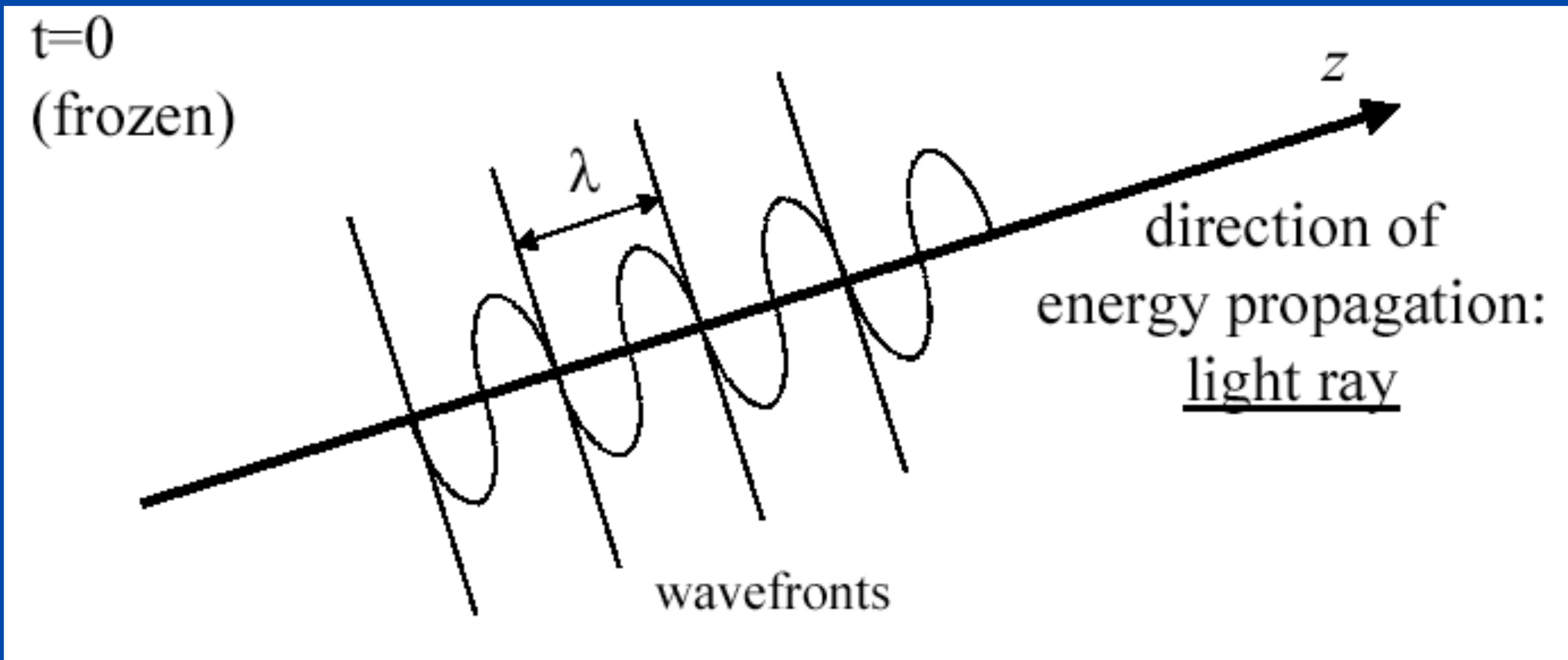
⇓

Quantum optics - photons, interaction with matter, lasers

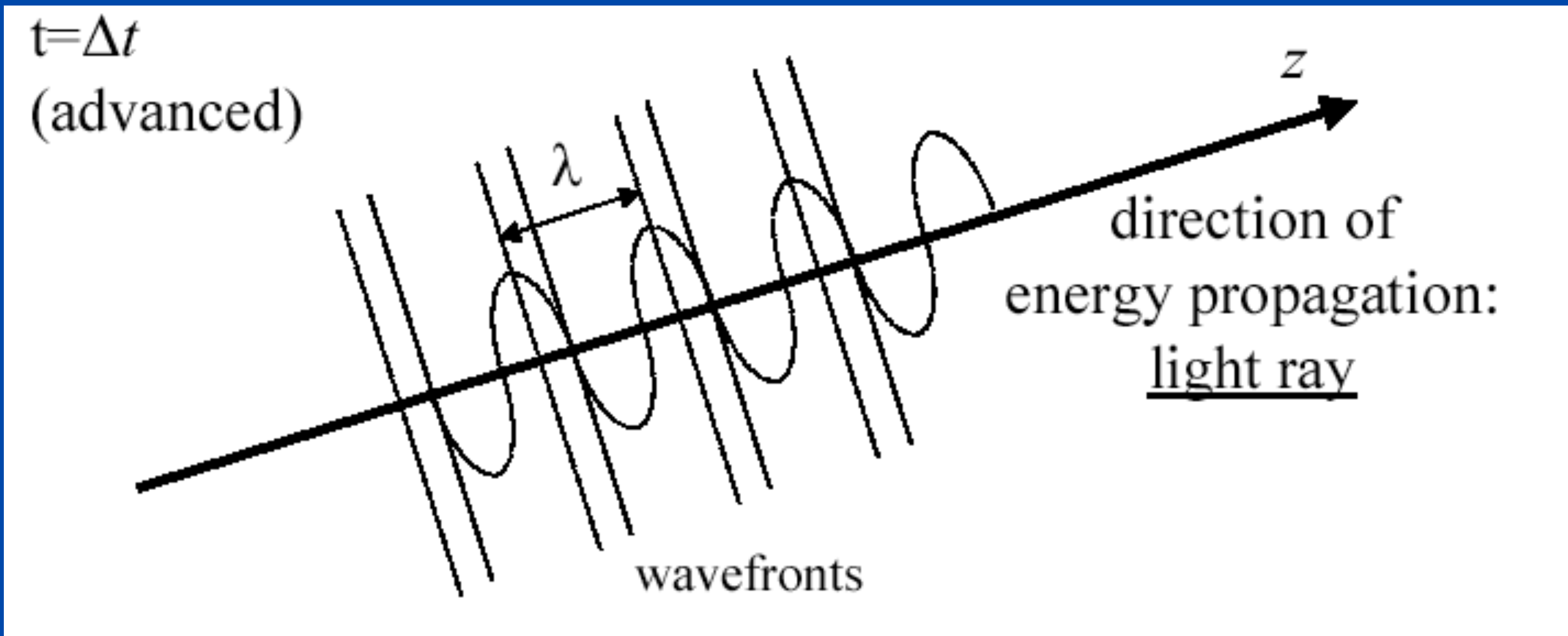
Review of geometrical optics: lenses, mirrors, and imaging

- Rays and wavefronts
- Laws of refraction and reflection
- Imaging
  - Pinhole camera
  - Lenses
  - Mirrors
- Diffraction limit (a heuristic derivation)

Rays and wavefronts

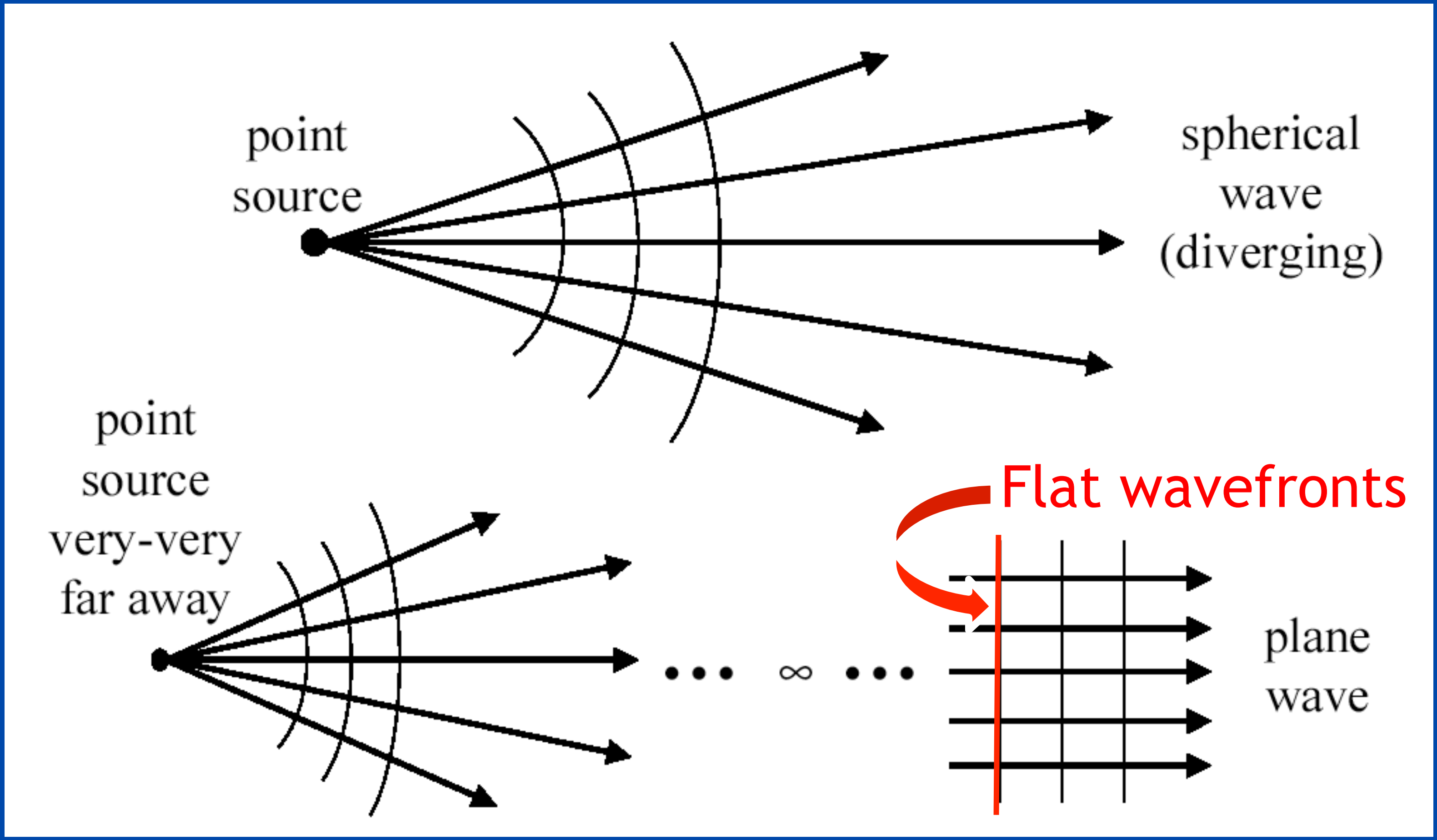


Rays and wavefronts



- In homogeneous media, light propagates in straight lines

Spherical waves and plane waves



Index of refraction: determines line propagation speed in a medium

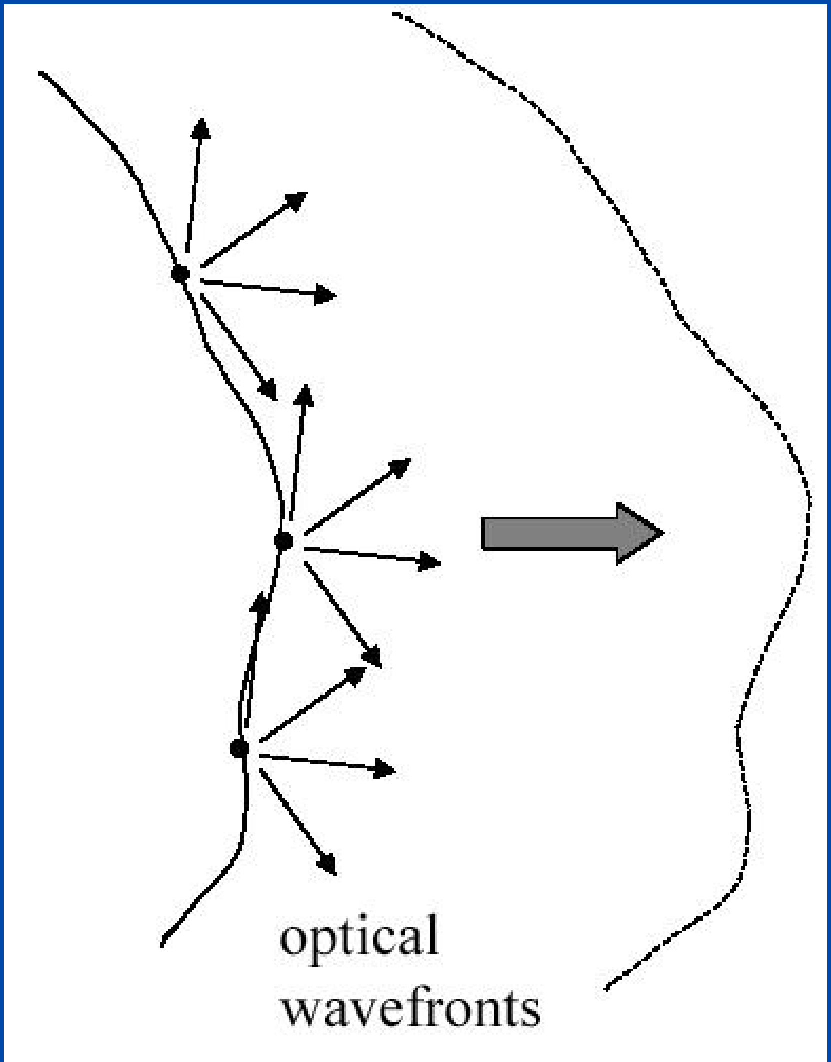
- Index of refraction n
- Phase velocity $v_\phi = \dfrac{c}{n}$
  - Speed of sinusoidal phase maxima
- In solid media like glass, $n > 1 \quad \Rightarrow \quad v_\phi < c$

Examples of index of refraction in media

Substance	Index of Refraction
Air	1.00029
Water	1.31
Fused silica (SiO_2)	1.46
Crown glass	1.52
ZnSe (10.6 μm)	2.40

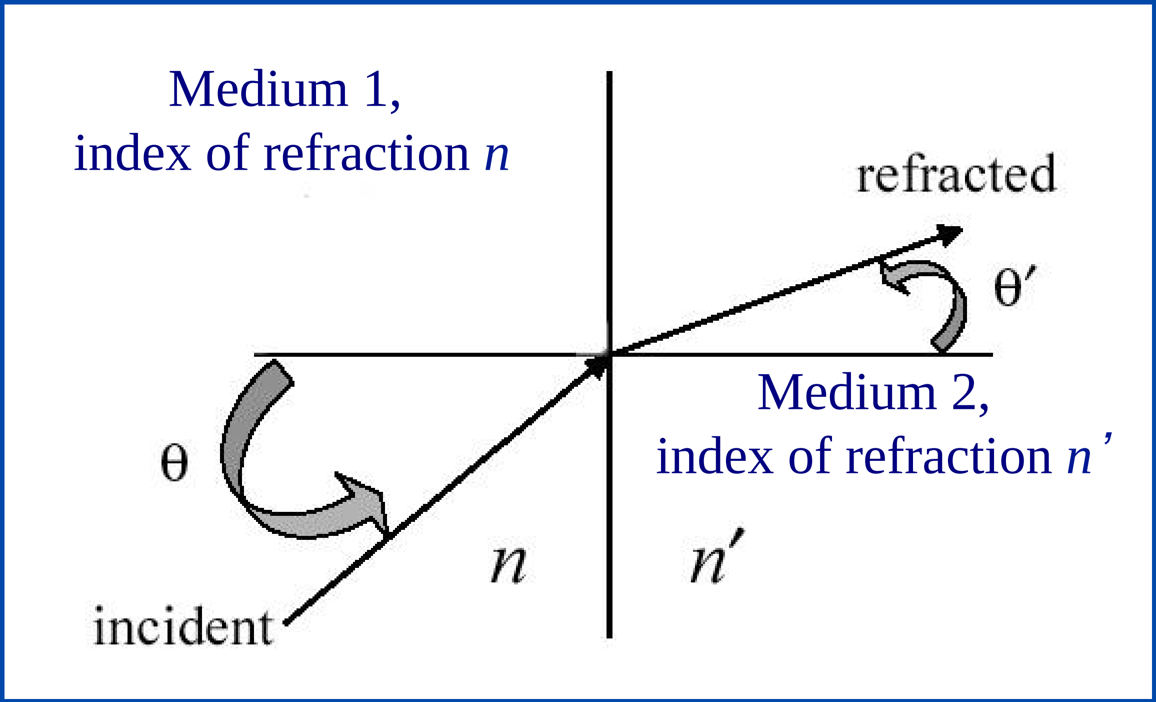
- Quite a large variation, even among common substances

Huygens' Principle



- Every point in a wavefront acts as a little secondary light source, and emits a spherical wave
- The propagating wave-front is the result of superposing all these little spherical waves
- Destructive interference in all but the direction of propagation

Refraction at a surface: Snell's Law



- Snell's law: $n \sin \vartheta = n' \sin \vartheta'$

The wave picture of refraction

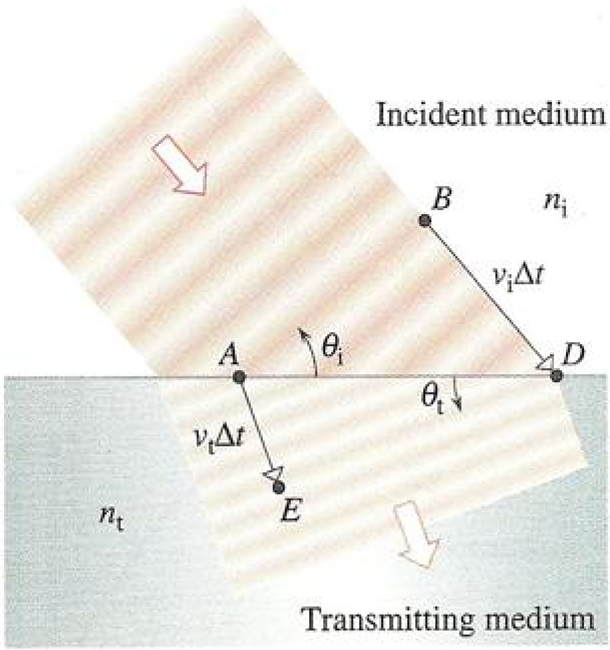


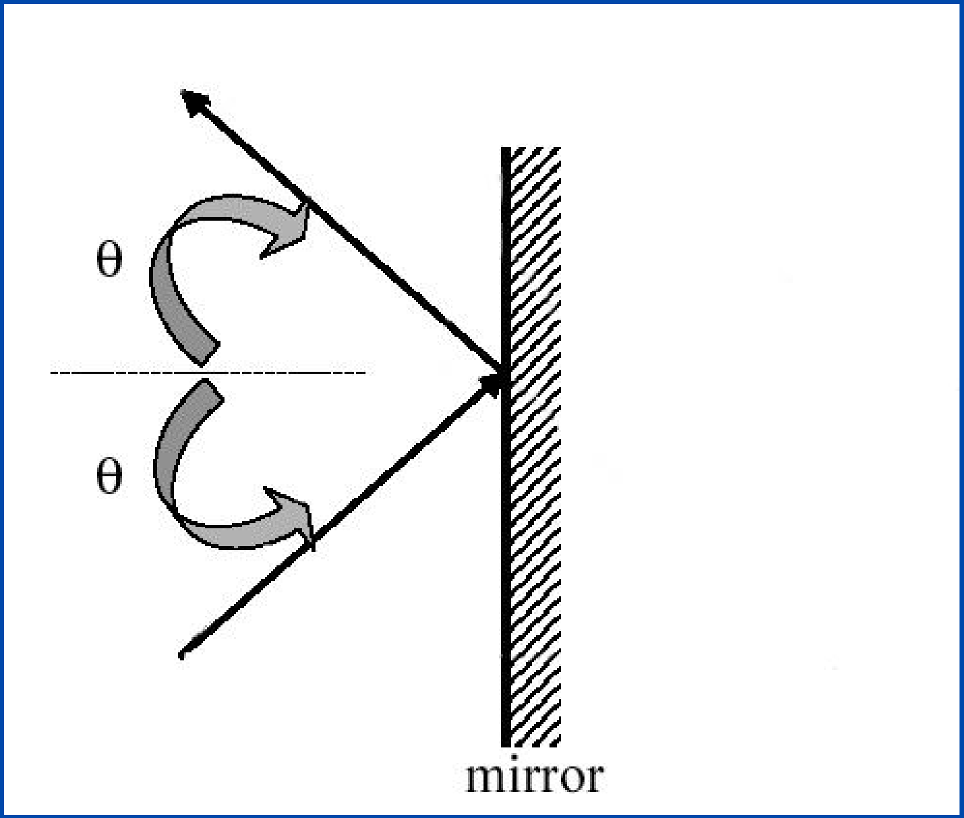
Figure 25.19 The wave picture of refraction. The atoms in the region of the surface of the transmitting medium reradiate wavelets that combine constructively to form a refracted beam.

- If $n_t > n_i$, phase velocity is slower in the transmitting medium
- Distance propagated in time Δt is shorter in transmitting medium

$$n_i \sin \vartheta_i = n_t \sin \vartheta_t$$

- Credit: Hecht

Reflection at a surface



- Angle of incidence equals angle of reflection

The wave picture of reflection

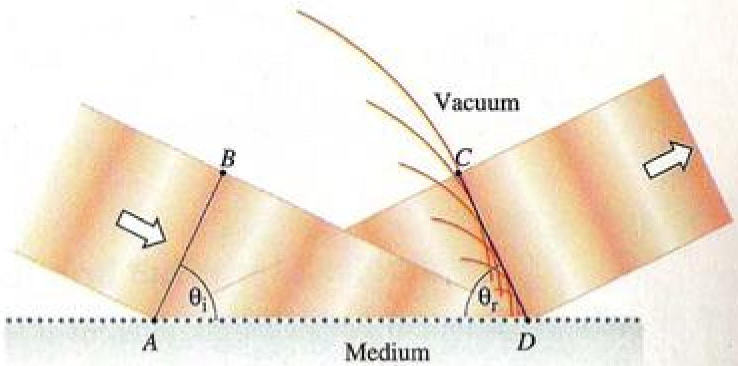


Figure 25.10 Wavefront geometry for reflection. The reflected wavefront $\overline{CD}$ is formed of waves scattered by the atoms on the surface from *A* to *D*. Just as the first wavelet arrives at *C* from *A*, the atom at *D* emits, and the wavefront along $\overline{CD}$ is completed.

- Atoms at surface re-radiate the EM fields
- The re-radiated waves undergo destructive interference, except in direction where $\theta_i = \theta_r$

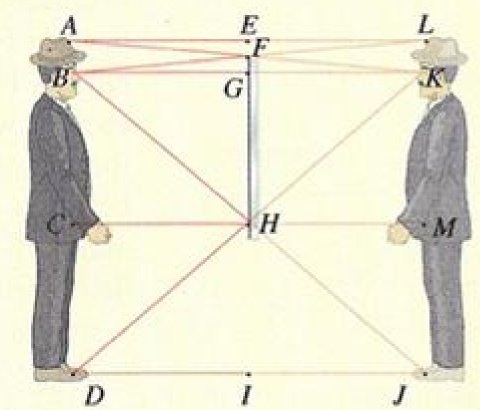
- Credit: Hecht

Concept Question

- You want to buy a full-length mirror for your bedroom, but they are all too expensive
- What is the length of the smallest vertical planar mirror in which you can see your entire standing body all at once?
- How should it be positioned?
- Hint:
- Draw a picture, and use similar triangles

Concept Question

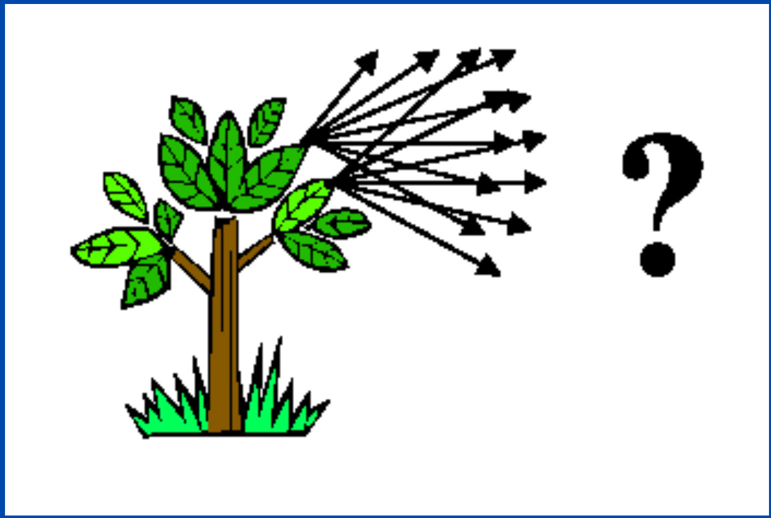
- You want to buy a full-length mirror for your bedroom, but they are all too expensive
- What is the length of the smallest vertical planar mirror in which you can see your entire standing body all at once?
- How should it be positioned?



A mirror half your height with its upper edge lowered by half the distance between your eye and the top of your head serves nicely.

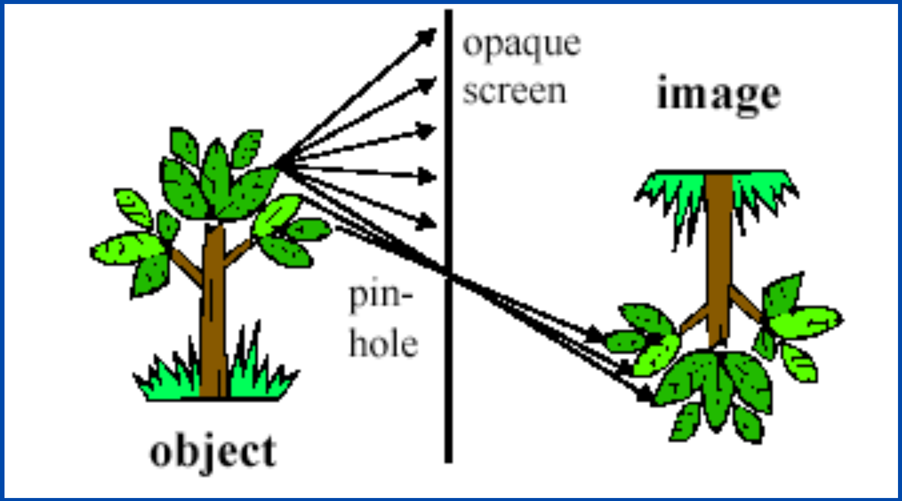
Why are imaging systems needed?

- Every point in the object scatters an incident light into a spherical wave
- The spherical waves from all the points on the object's surface get mixed together as they propagate toward you
- An imaging system reassigns (focuses) all the rays from a single point on the object onto another point in space (the "focal point"), so you can distinguish details of the object.

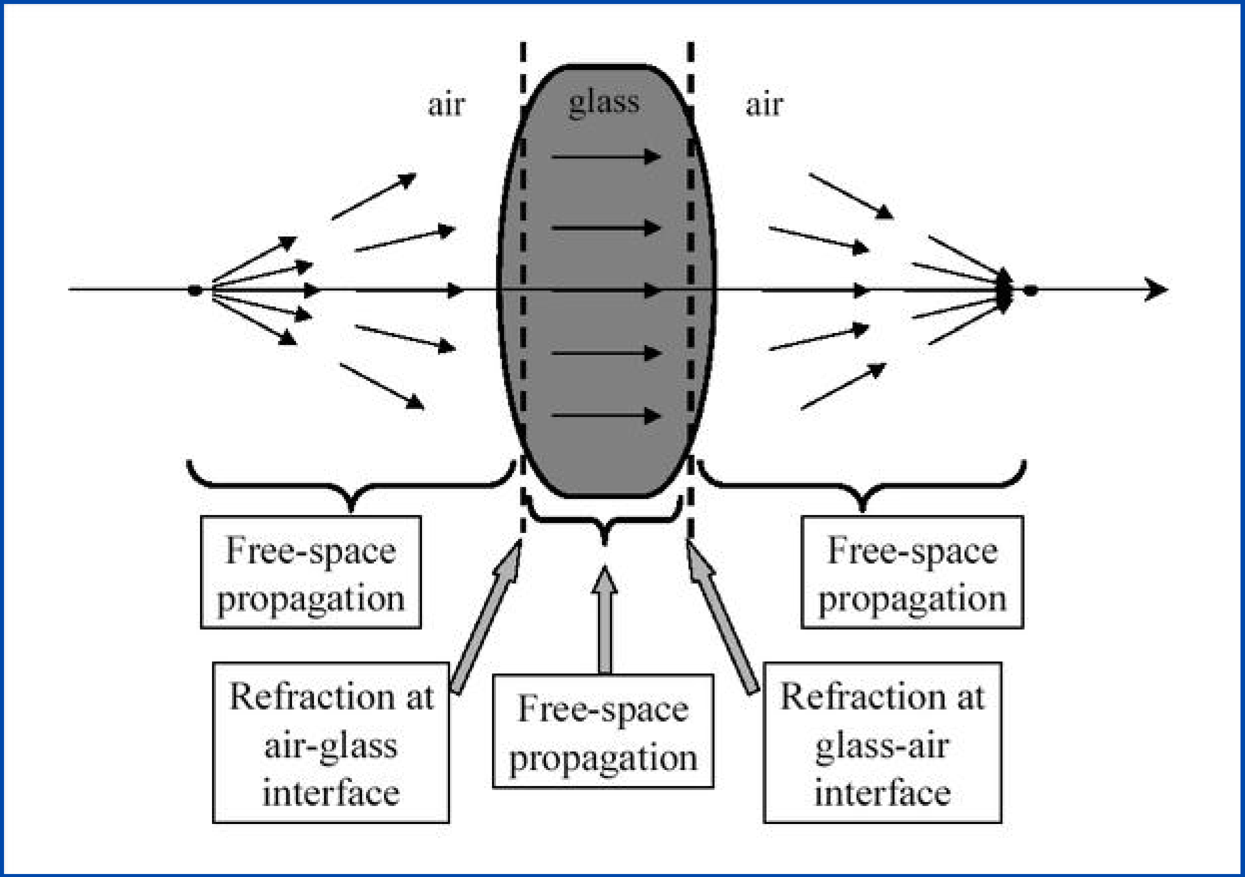


Pinhole camera is simplest imaging instrument

- Opaque screen with a pinhole blocks all but one ray per object point from reaching the image space.
- An image is formed (upside down). Good news.
- BUT most of the light is wasted (it is stopped by the opaque sheet). Bad news.
- Also, diffraction of light as it passes through the small pinhole produces artifacts in the image.



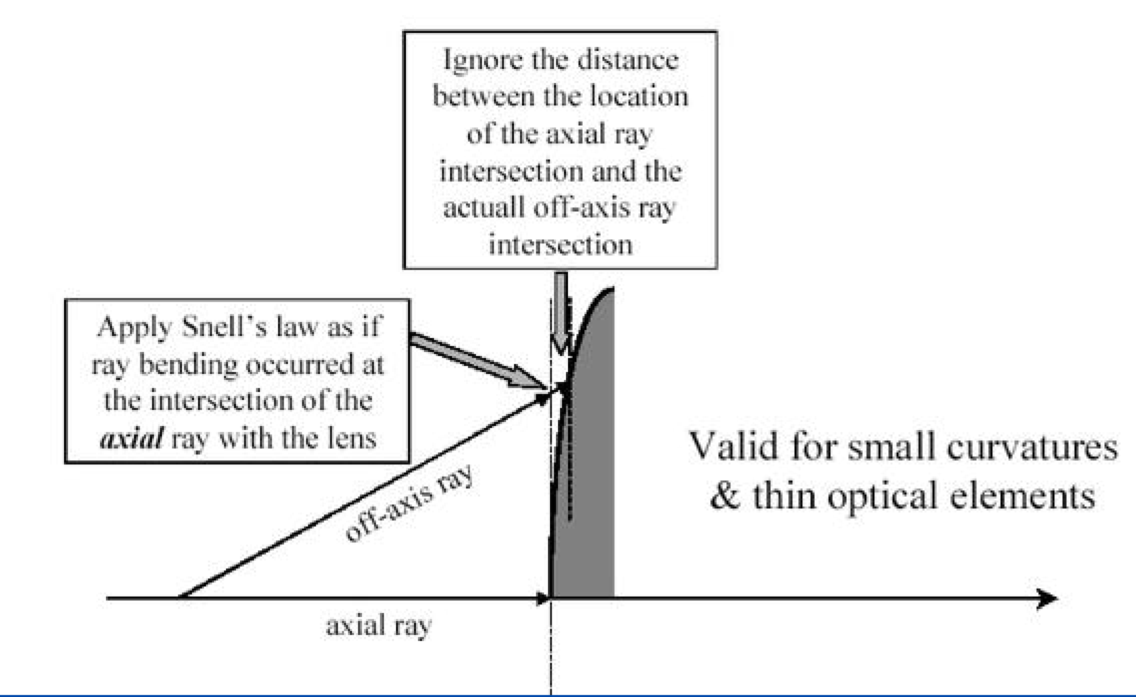
Imaging with lenses: doesn't throw away as much light as pinhole camera



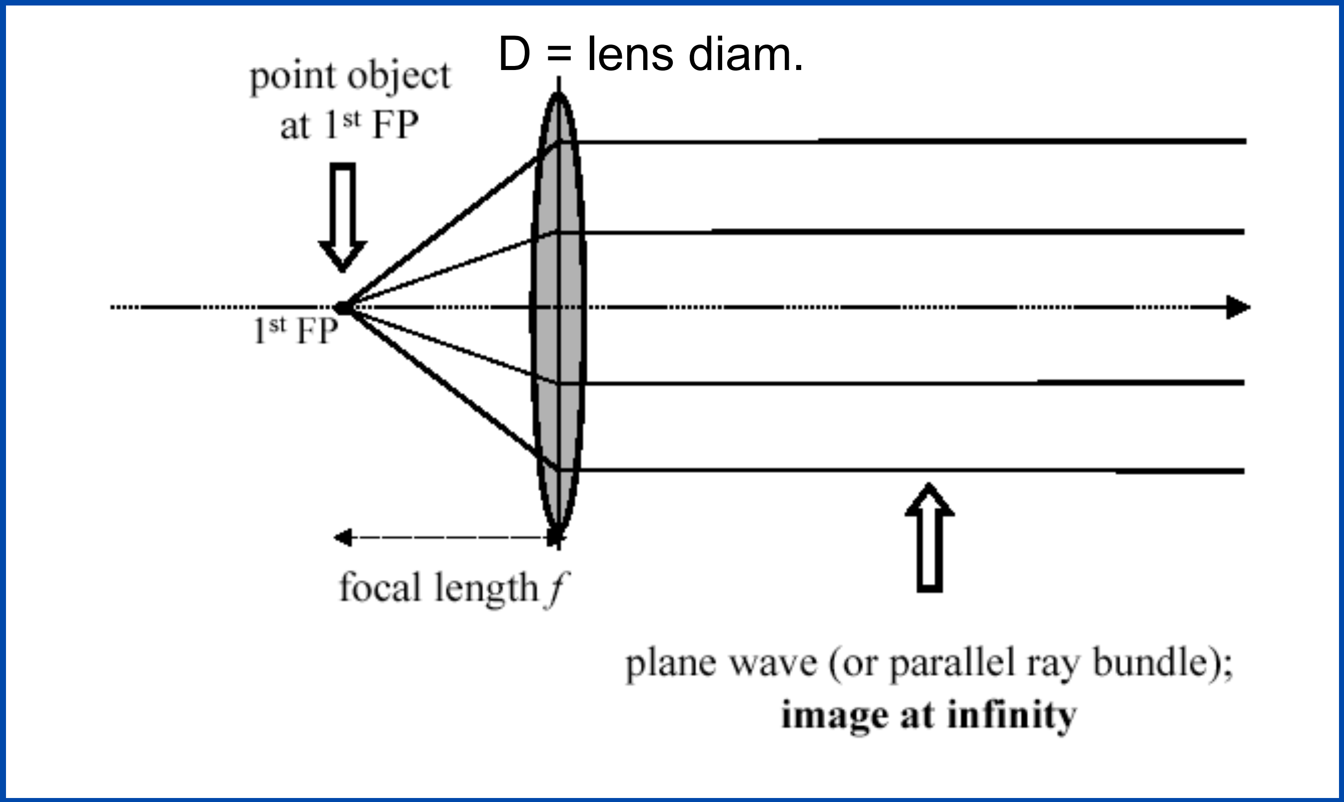
Collects <u>all</u> rays that pass through solid-angle of lens

"Paraxial approximation" or "first order optics" or "Gaussian optics"

- Angle of rays with respect to optical axis is small
- First-order Taylor expansions:
  - $\sin\theta \sim \tan\theta \sim \theta$, $\cos\theta \sim 1$, $(1 + x)^{1/2} \sim 1 + x / 2$

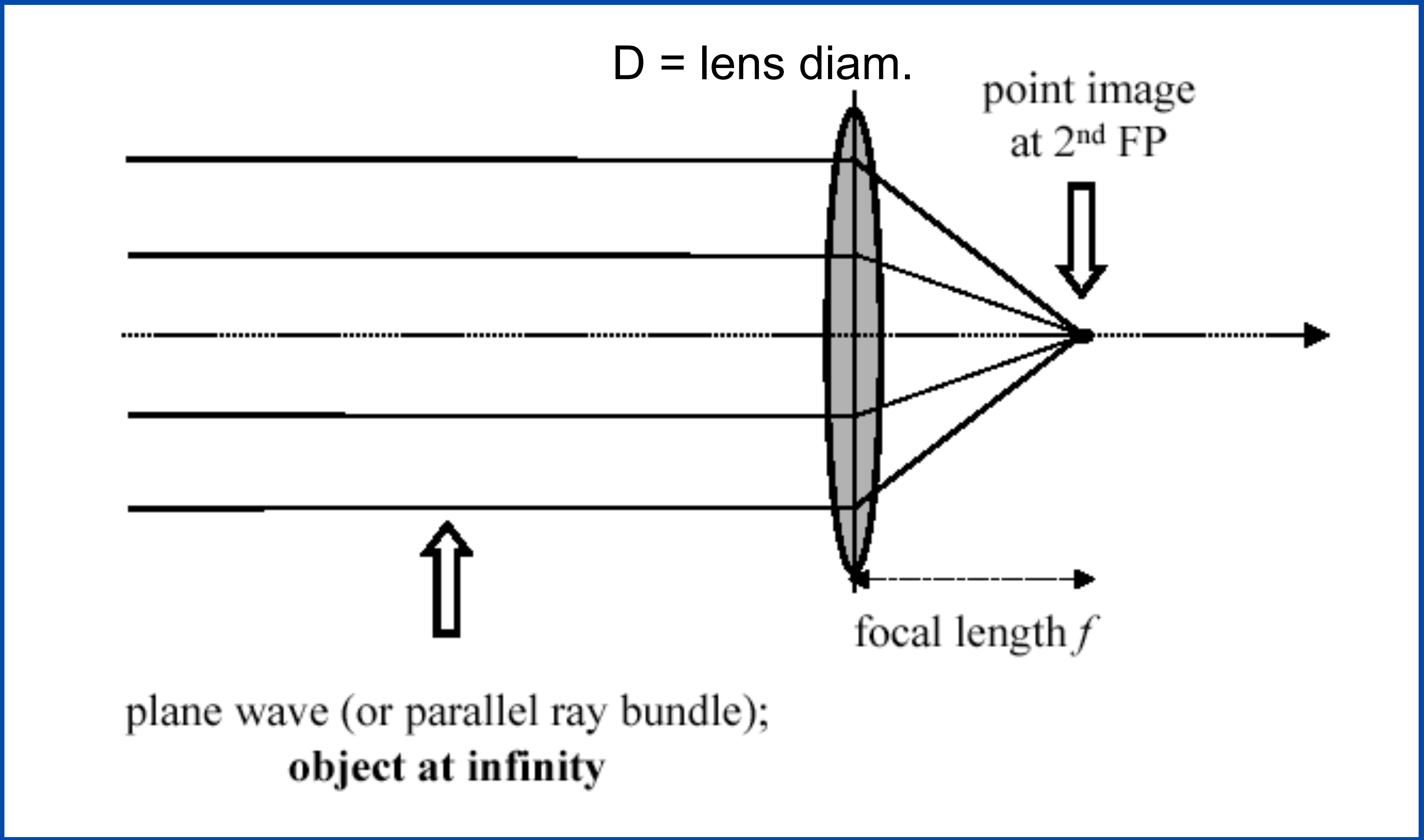


Thin lenses, part 1



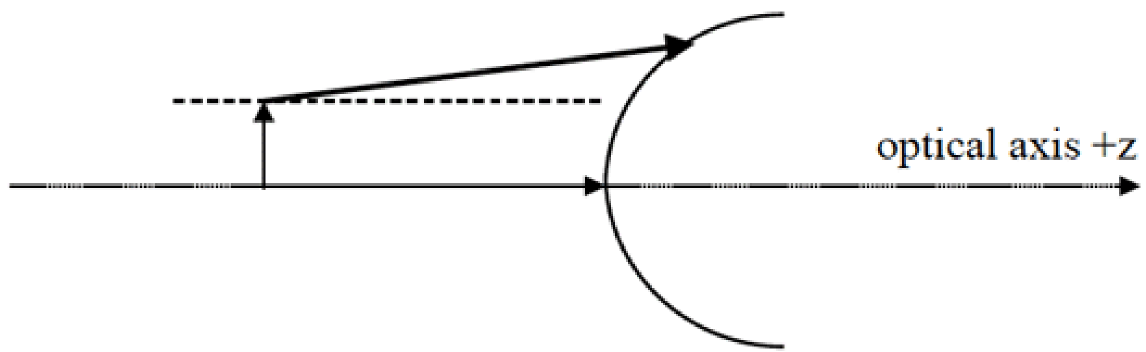
Definition: f-number: $f / \# = f / D$

Thin lenses, part 2



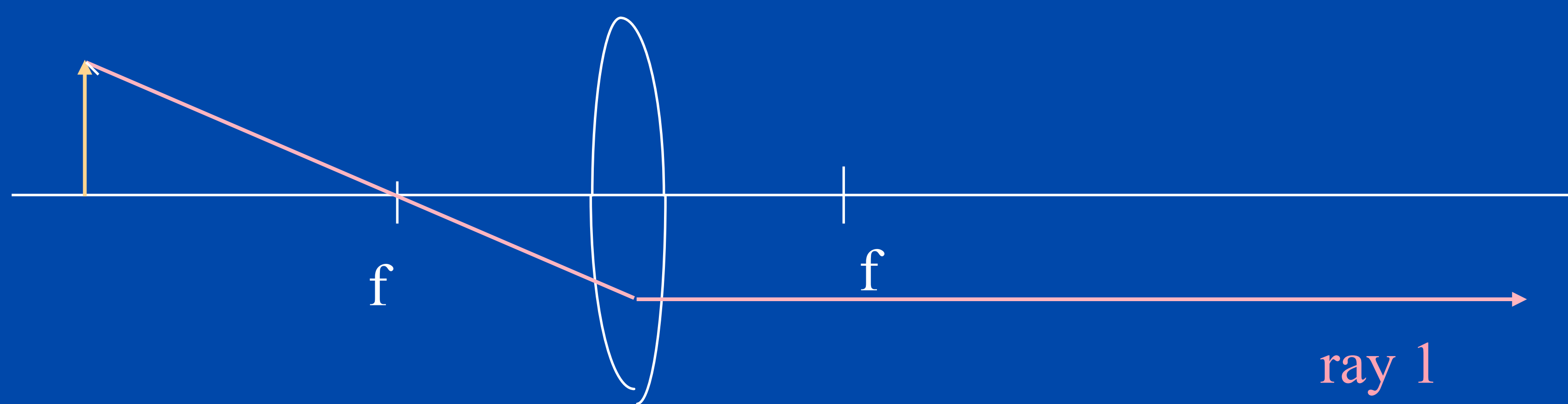
Sign conventions for refraction

- Light travels from left to right
- A radius of curvature is positive if the surface is convex towards the left
- Longitudinal distances are positive if pointing to the right
- Lateral distances are positive if pointing up
- Ray angles are positive if the ray direction is obtained by rotating the +z axis counterclockwise through an acute angle



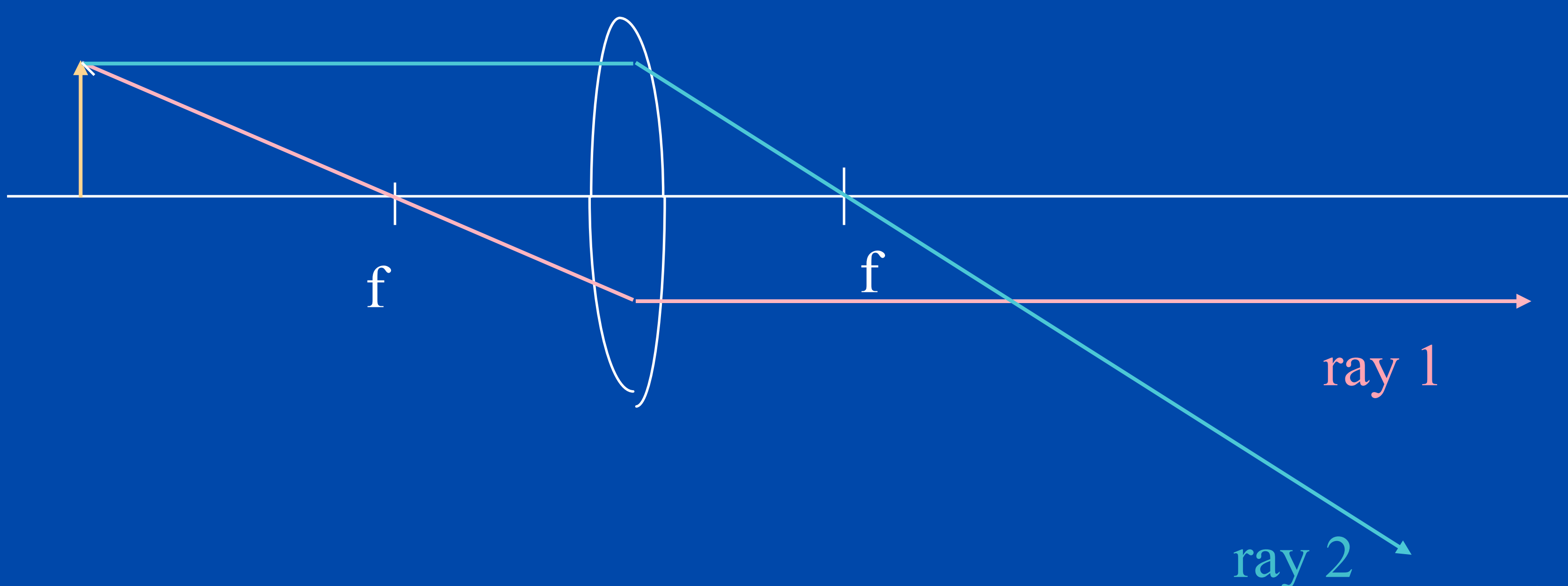
Refraction and the Lens-users Equation

- Any ray that goes through the focal point on its way to the lens will come out parallel to the optical axis. (ray 1)



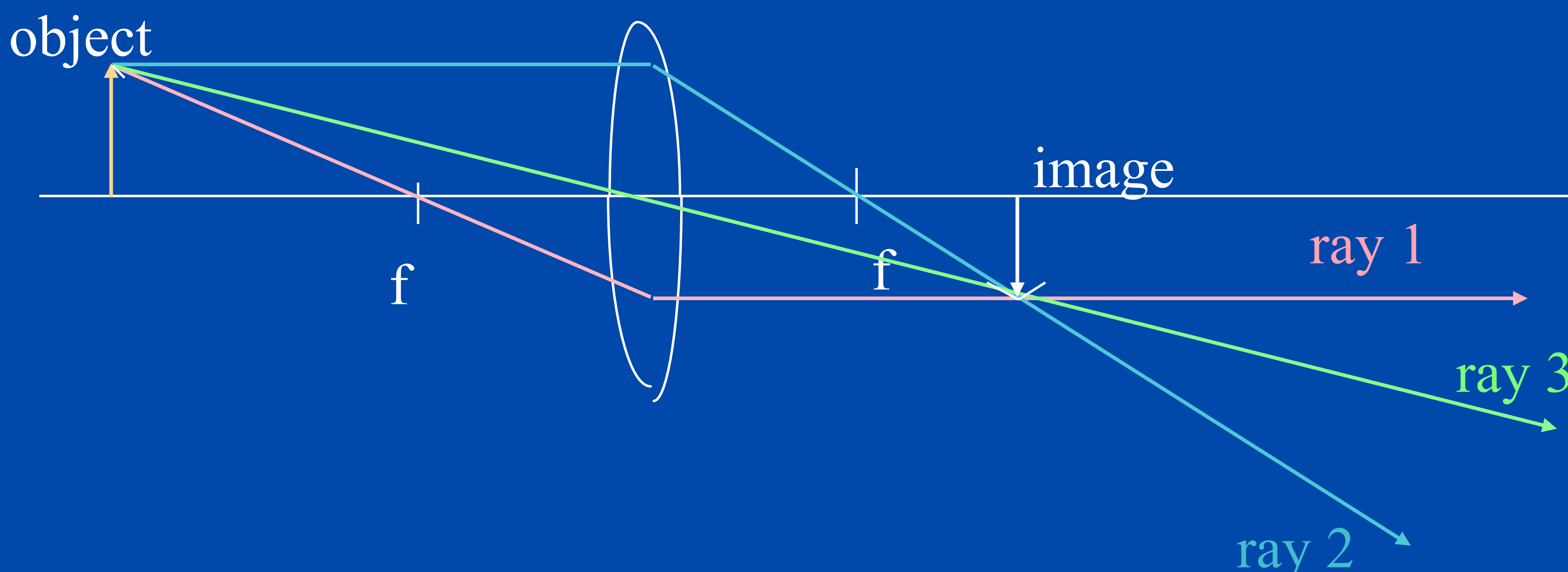
Refraction and the Lens-users Equation

- Any ray that goes through the focal point on its way from the lens, must go into the lens parallel to the optical axis. (ray 2)



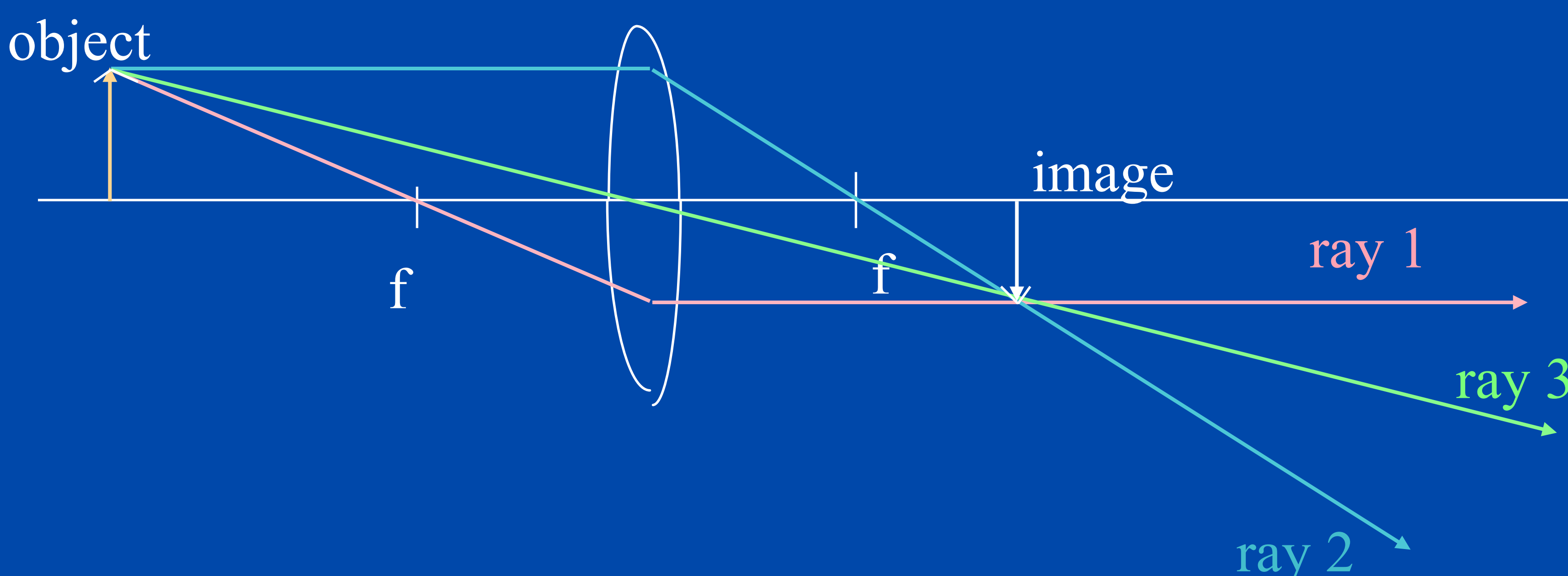
Refraction and the Lens-users Equation

- Any ray that goes through the center of the lens must go essentially undeflected. (ray 3)



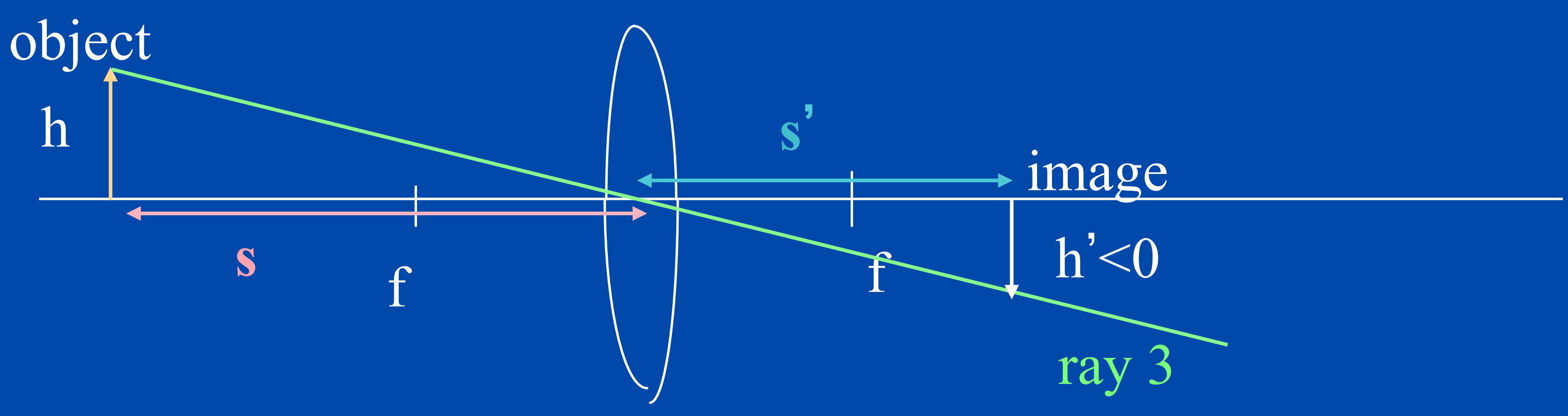
Refraction and the Lens-users Equation

- Note that a real image is formed (image is on opposite side of the lens from the object)
- Note that the image is up-side-down.



Refraction and the Lens-users Equation

- By looking at ray 3 alone, we can see by similar triangles that $M = h'/h = -s'/s$

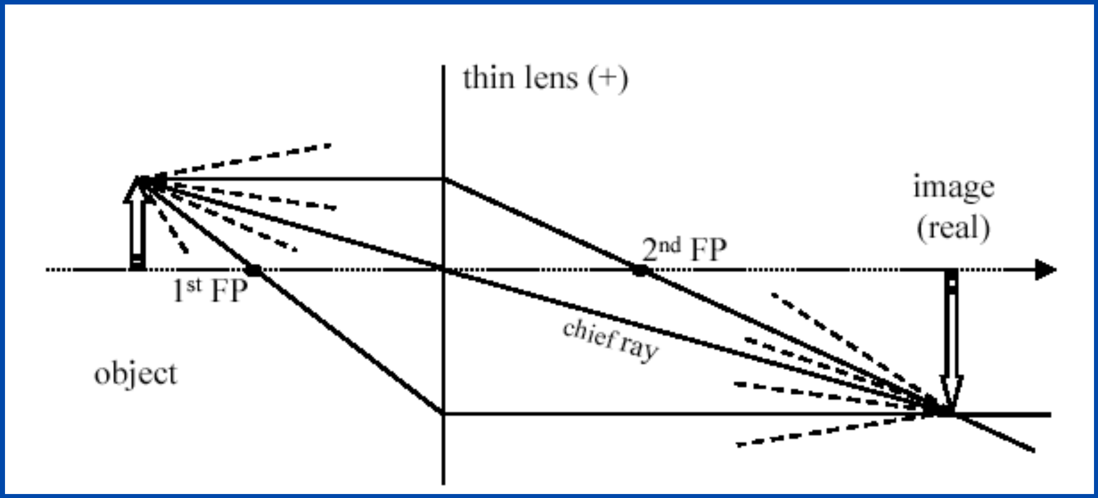


Example: f = 10 cm; s = 40 cm; s' = 13.3 cm:

M = -13.3/40 = -0.33

Note h' is up-side-down and so is < 0

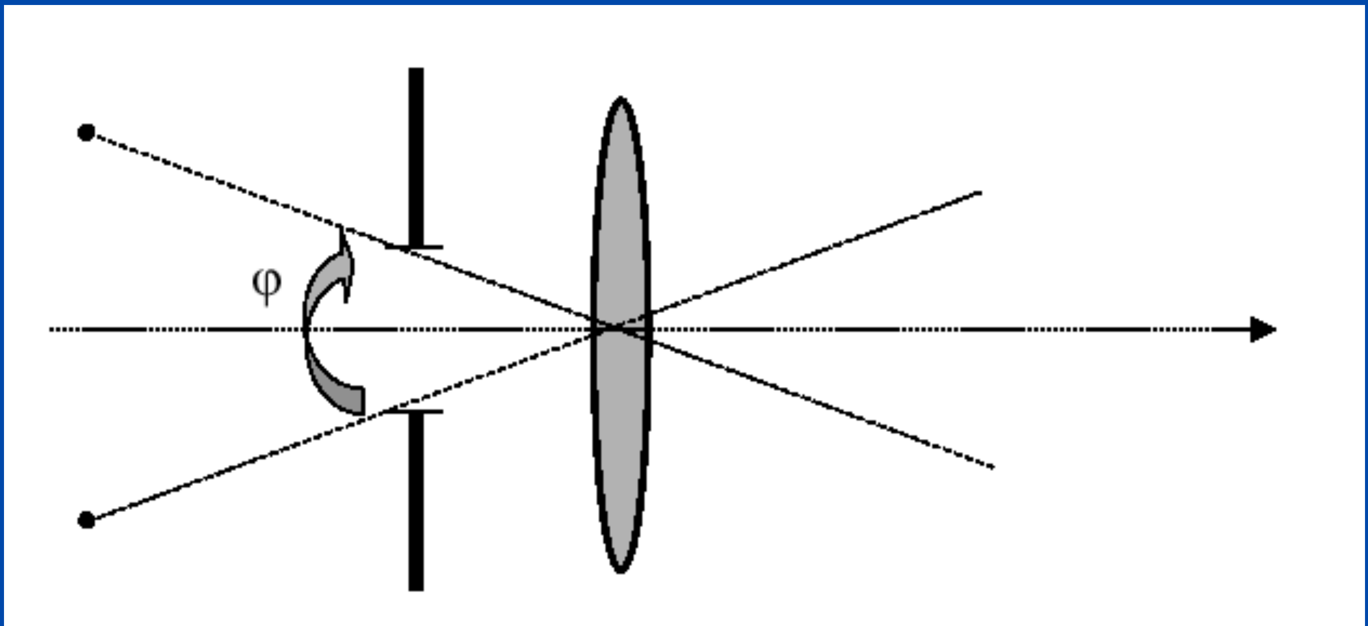
Ray-tracing with a thin lens



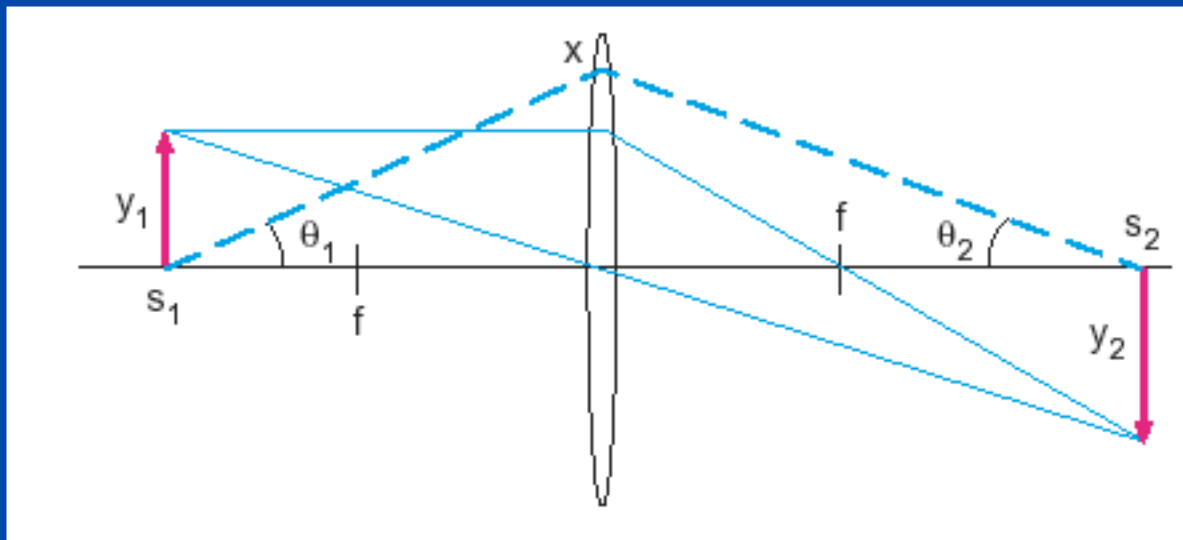
- Image point (focus) is located at intersection of ALL rays passing through the lens from the corresponding object point
- Easiest way to see this: trace rays passing through the two foci, and through the center of the lens (the "chief ray") and the edges of the lens

Definition: Field of view (FOV) of an imaging system

- Angle that the "chief ray" from an object can subtend, given the pupil (entrance aperture) of the imaging system
- Recall that the chief ray propagates through the lens un-deviated

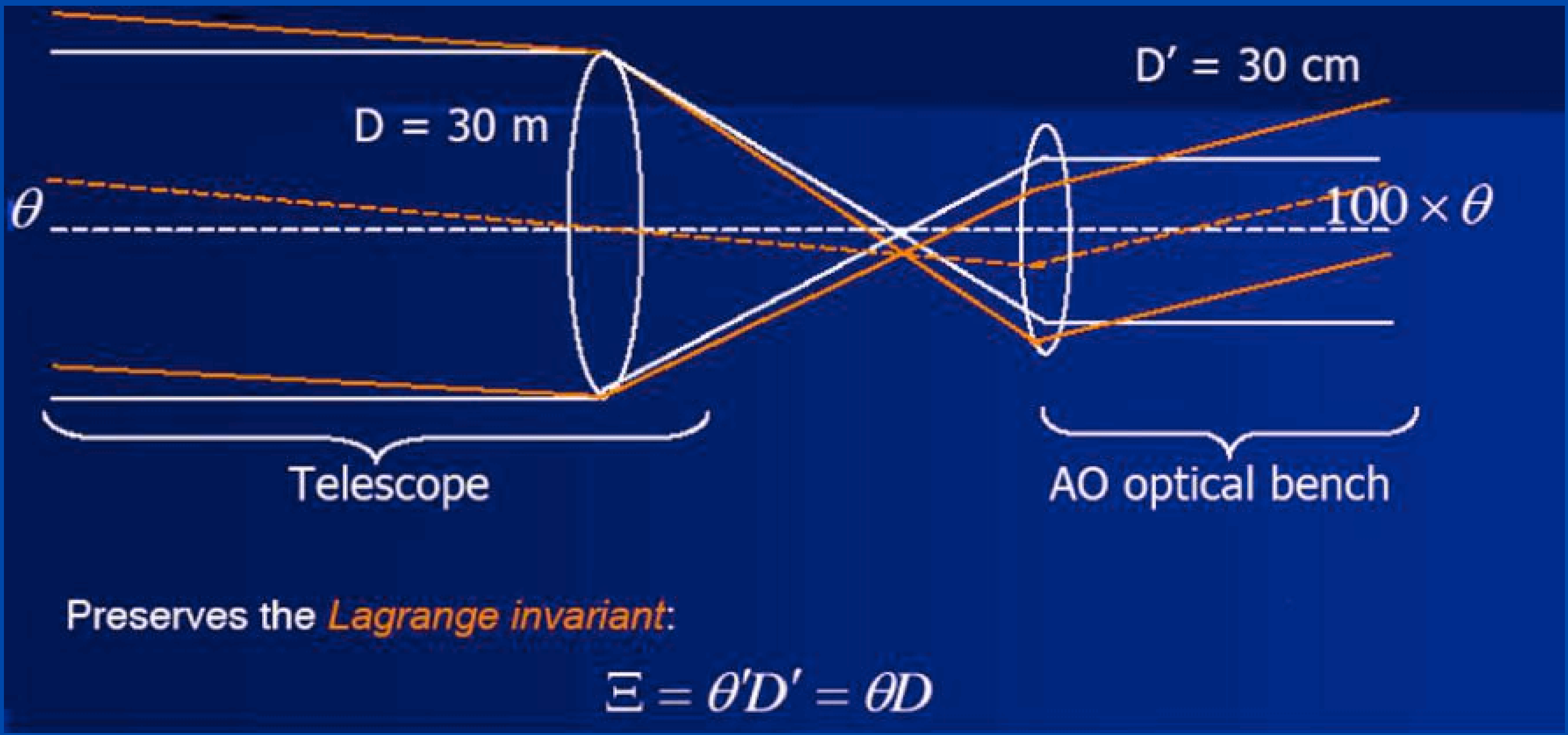


Optical invariant (= Lagrange invariant)



$$y_1\vartheta_1 = y_2\vartheta_2$$

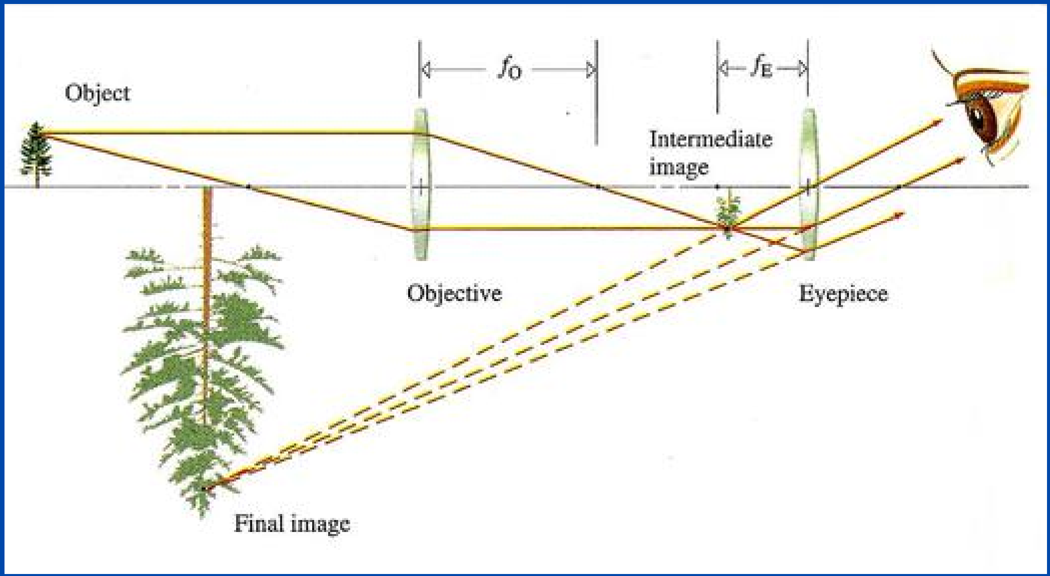
Lagrange invariant has important consequences for AO on large telescopes



- Deformable mirror is much smaller than primary mirror
- Hence angles within AO system are much larger
- Consequences: limitations on field of view; vignetting

From Don Gavel

Refracting telescope: <u>two</u> lenses whose focal points coincide

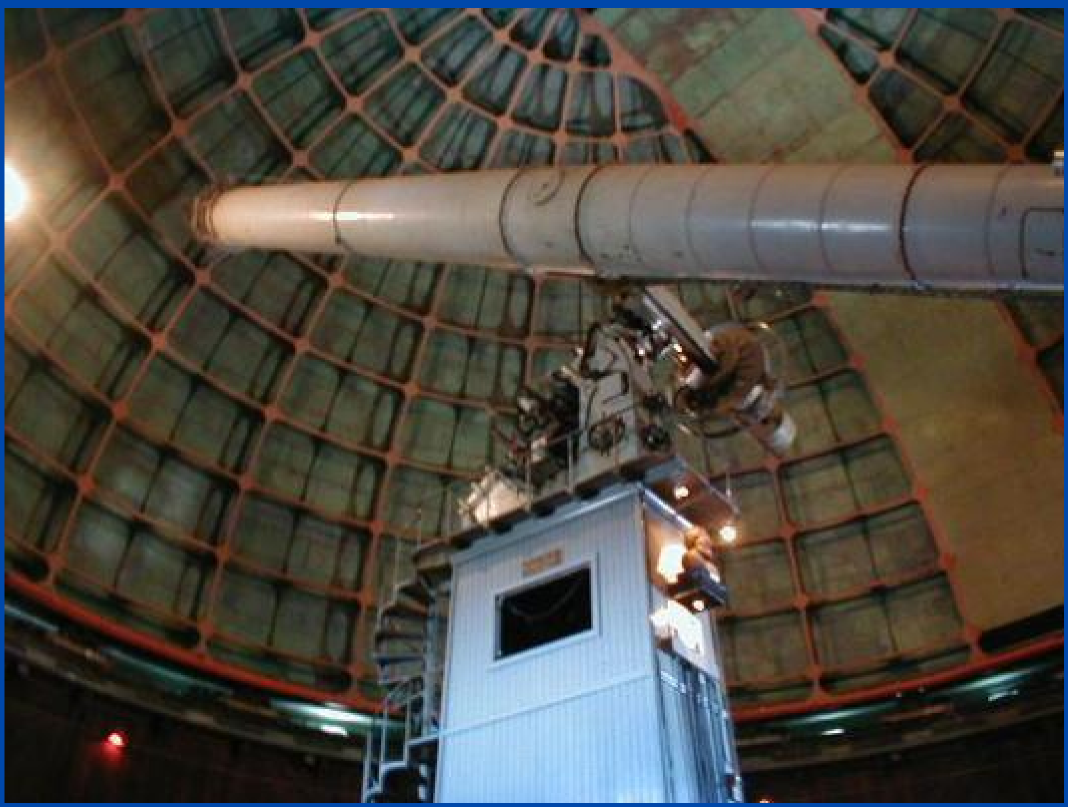


$$\frac{1}{f_{obj}} = \frac{1}{s_0} + \frac{1}{s_1} \approx \frac{1}{s_1} \text{ since } s_0 \to \infty$$

$$\text{so } s_1 \approx f_{obj}$$

- Main point of telescope: to gather more light than eye. Secondarily, to magnify image of the object
- Magnifying power $M_{tot} = -\, f_{Objective} \,/\, f_{Eyepiece}$ so for high magnification, make $f_{Objective}$ as large as possible (long tube) and make $f_{Eyepiece}$ as short as possible

Lick Observatory's 36" Refractor: one long telescope!



Concept Question

- Give an intuitive explanation for why the magnifying power of a refracting telescope is

$$M_{tot} = - f_{Objective} / f_{Eyepiece}$$

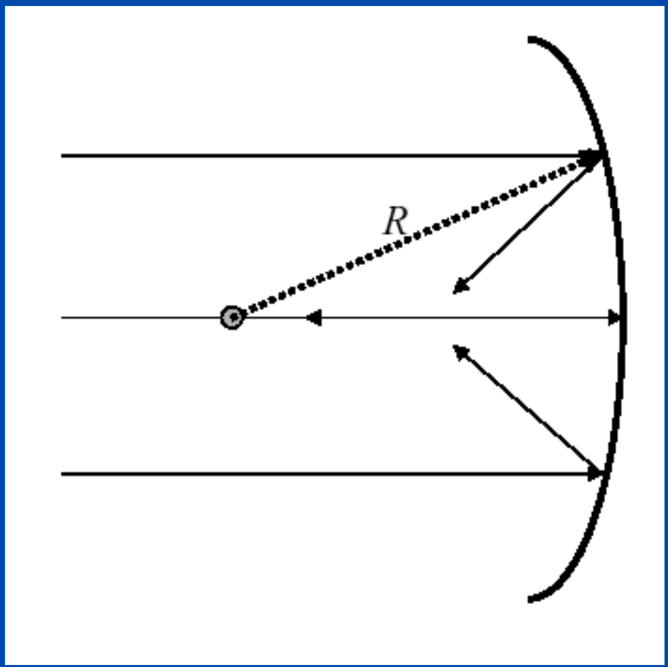
Make sketches to illustrate your reasoning

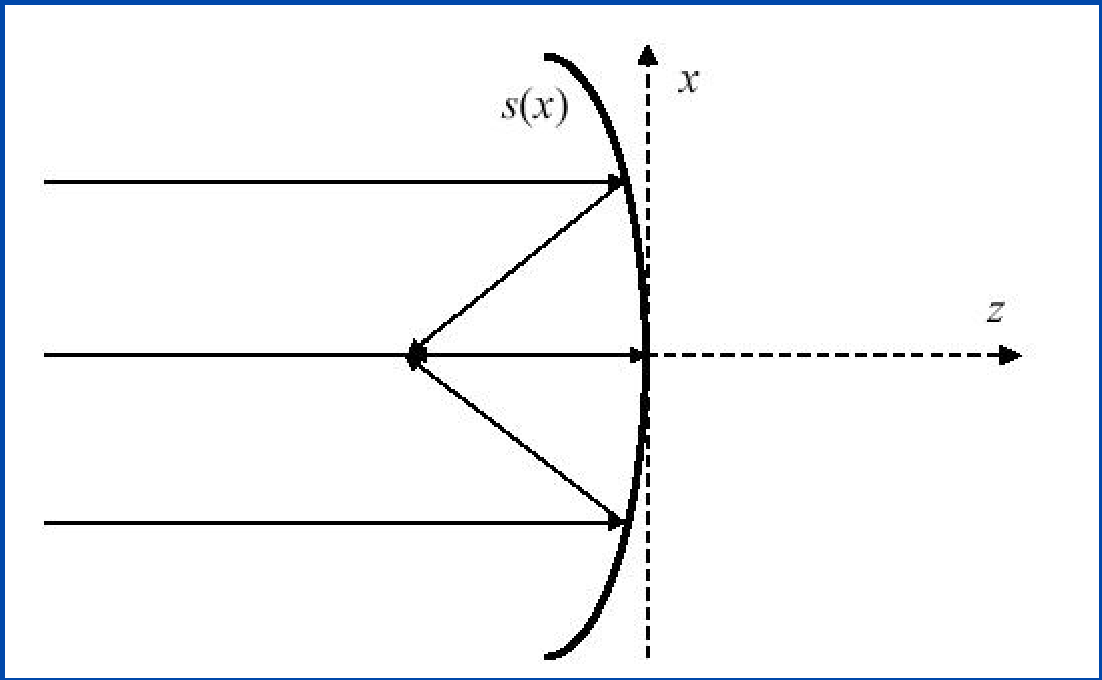
Time for a short break

- Please get up and move around!

Imaging with mirrors: spherical and parabolic mirrors

$f = -R/2$





<u>Spherical surface</u>: in paraxial approx, focuses incoming parallel rays to (approx) a point

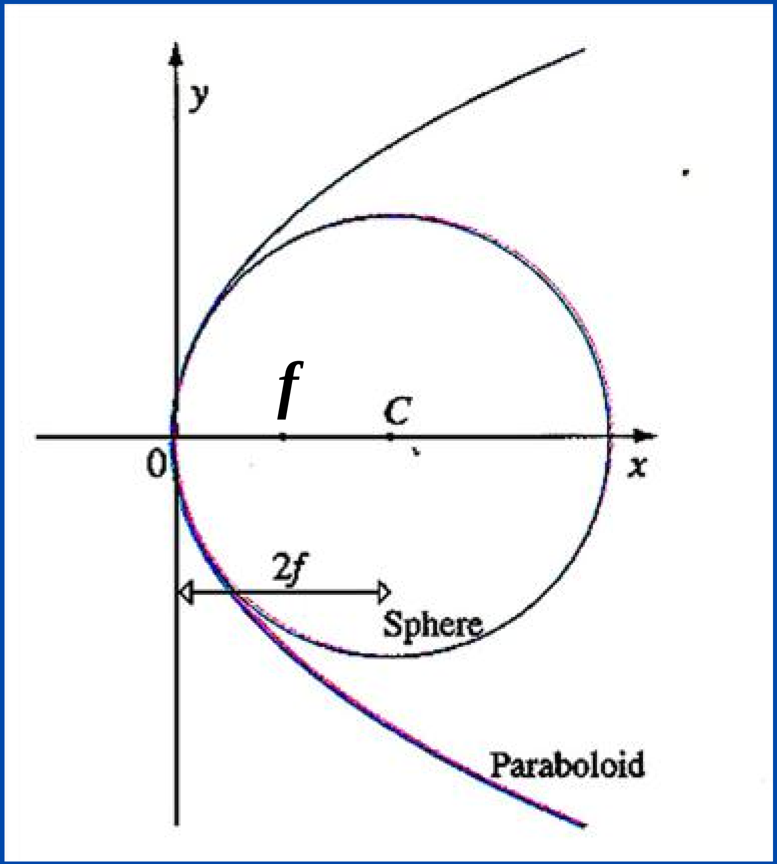
<u>Parabolic surface</u>: perfect focusing for parallel rays (e.g. satellite dish, radio telescope)

Problems with spherical mirrors

- Optical aberrations (mostly spherical aberration and coma)
  - Especially if *f*-number is small ("fast" focal ratio, short telescope, big angles)

Focal length of mirrors

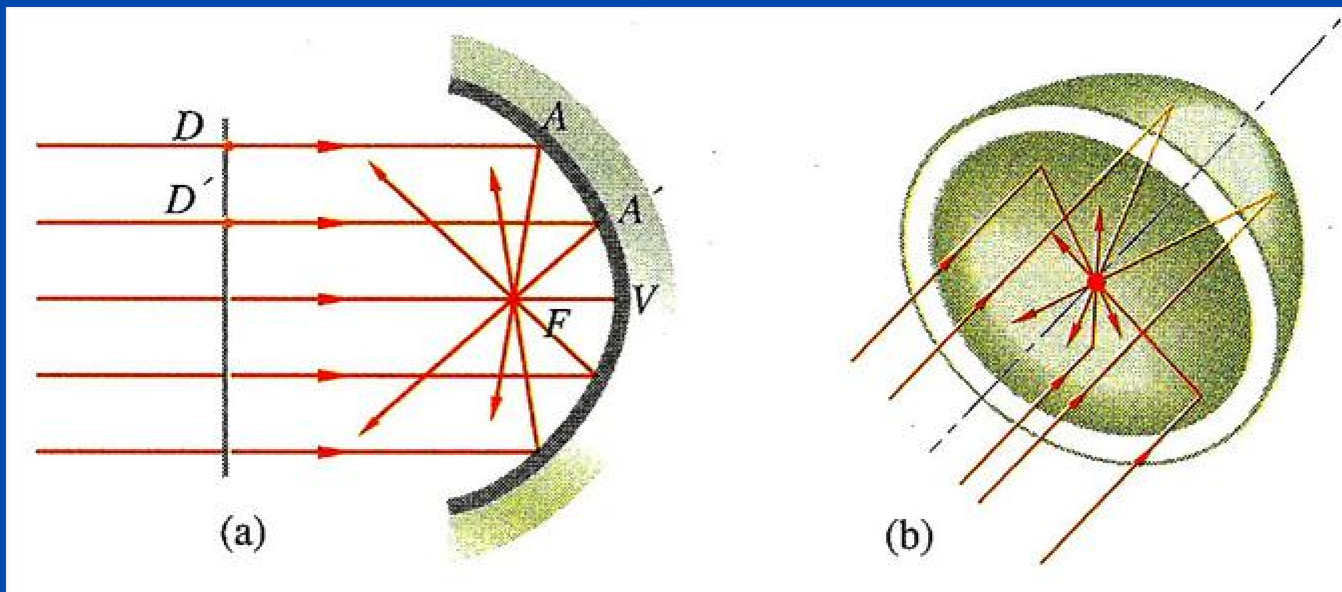
- Focal length of spherical mirror is $f_{sp} = - R/2$
- Convention: f is positive if it is to the left of the mirror
- Near the optical axis, parabola and sphere are very similar, so that $f_{par} = - R/2$ as well.



Sign conventions for reflection

- Light travels from left to right *before reflection and from right to left after reflection*
- A radius of curvature is positive if the surface is convex towards the left
- Longitudinal distances *before reflection* are positive if pointing to the right; *longitudinal distances after reflection are positive if pointing to the left*
- Longitudinal distances are positive if pointing up
- Ray angles are positive if the ray direction is obtained by rotating the +z axis counterclockwise through an acute angle

Parabolic mirror: focus in 3D



Mirror equations

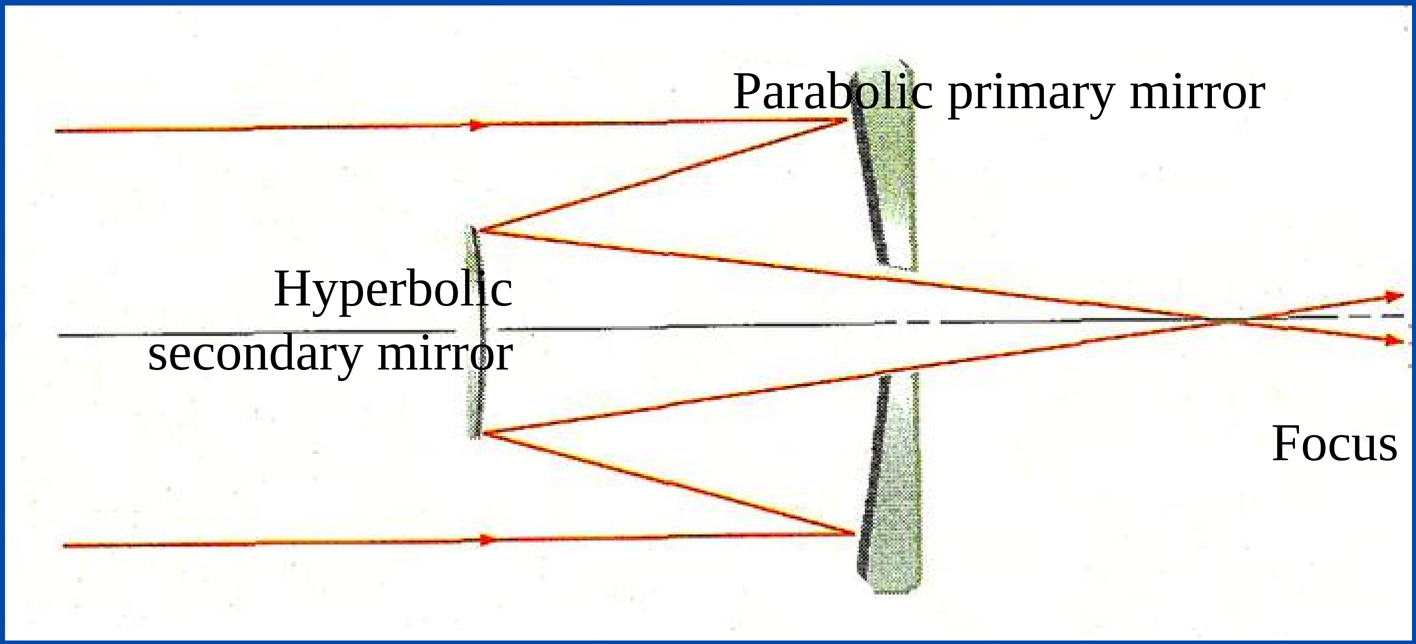
- Imaging condition for spherical mirror $$\frac{1}{s_0} + \frac{1}{s_1} = -\frac{2}{R}$$

- Focal length $$f = -\frac{R}{2}$$

- Magnifications $$M_{transverse} = -\frac{s_0}{s_1}$$

$$M_{angle} = -\frac{s_1}{s_0}$$

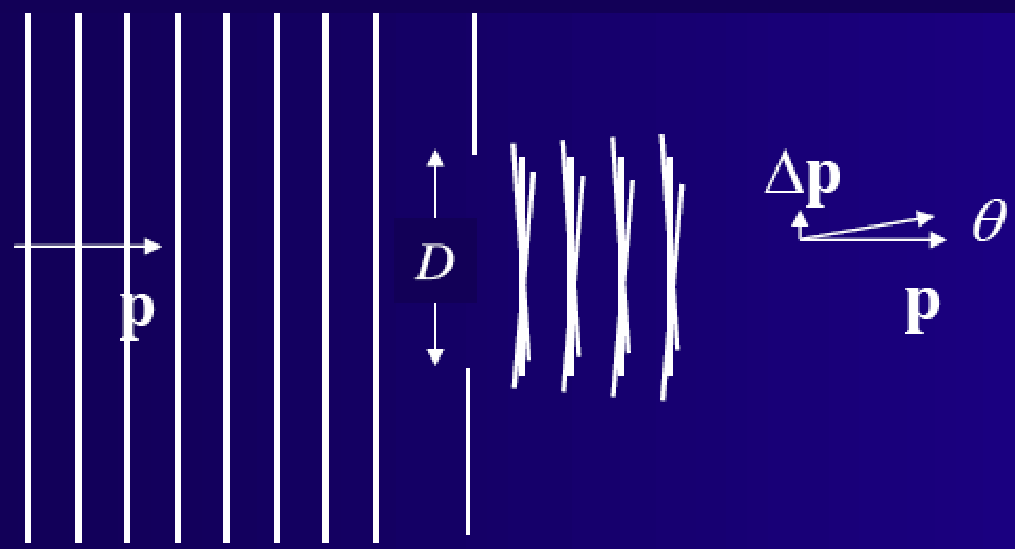
Cassegrain reflecting telescope



- Hyperbolic secondary mirror: 1) reduces off-axis aberrations, 2) shortens physical length of telescope.

- Can build mirrors with much shorter focal lengths than lenses. Example: 10-meter primary mirrors of Keck Telescopes have focal lengths of 17.5 meters (f/1.75). About same as Lick 36" refractor.

A look ahead to Fourier Optics: Heuristic derivation of the diffraction limit



Uncertainty principle

$$\Delta x \Delta p \cong h$$

Photon momentum

$$p = h/\lambda$$

$$\Delta \mathbf{x} = \infty$$

$$\Delta \mathbf{p} = 0$$

$$\Delta x = D$$

$$\Delta p = h/D$$

$$\Rightarrow$$

Law of diffraction

$$\theta \cong \Delta p / p = \lambda / D$$

Aberrations

- In optical systems
- In atmosphere
- Description in terms of Zernike polynomials

- Based on slides by Brian Bauman, LLNL and UCSC, and Gary Chanan, UCI

Third order aberrations

- *sin θ* terms in Snell's law can be expanded in power series

$$n \sin\theta = n' \sin\theta'$$

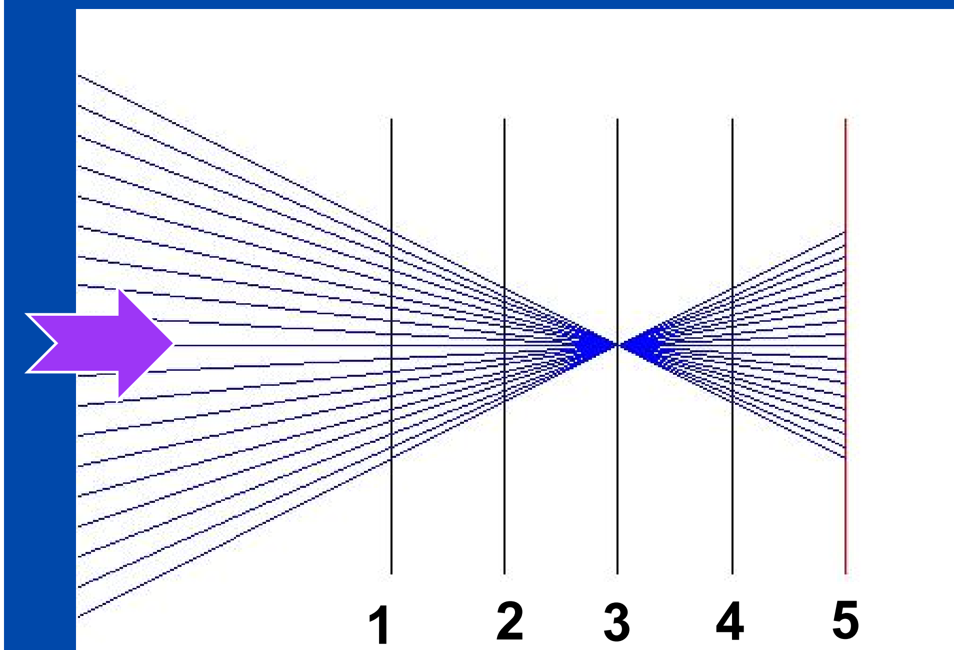
$$n\,(\theta - \theta^3/3! + \theta^5/5! + \ldots) = n'\,(\theta' - \theta'^3/3! + \theta'^5/5! + \ldots)$$

- Paraxial ray approximation: keep only θ terms (first order optics; rays propagate nearly along optical axis)
  - Piston, tilt, defocus
- Third order aberrations: result from adding θ^3 terms
  - Spherical aberration, coma, astigmatism, ...

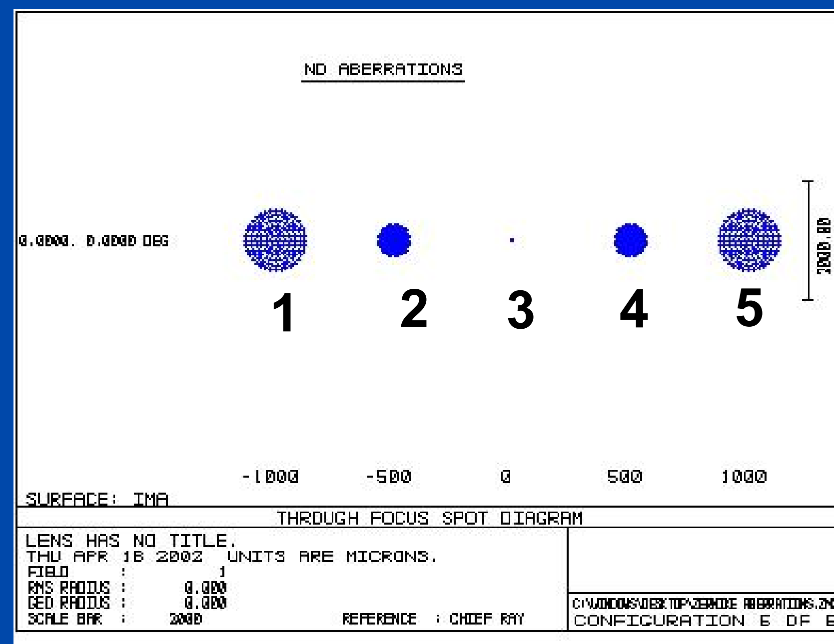
Different ways to illustrate optical aberrations

Side view of a fan of rays

(No aberrations)

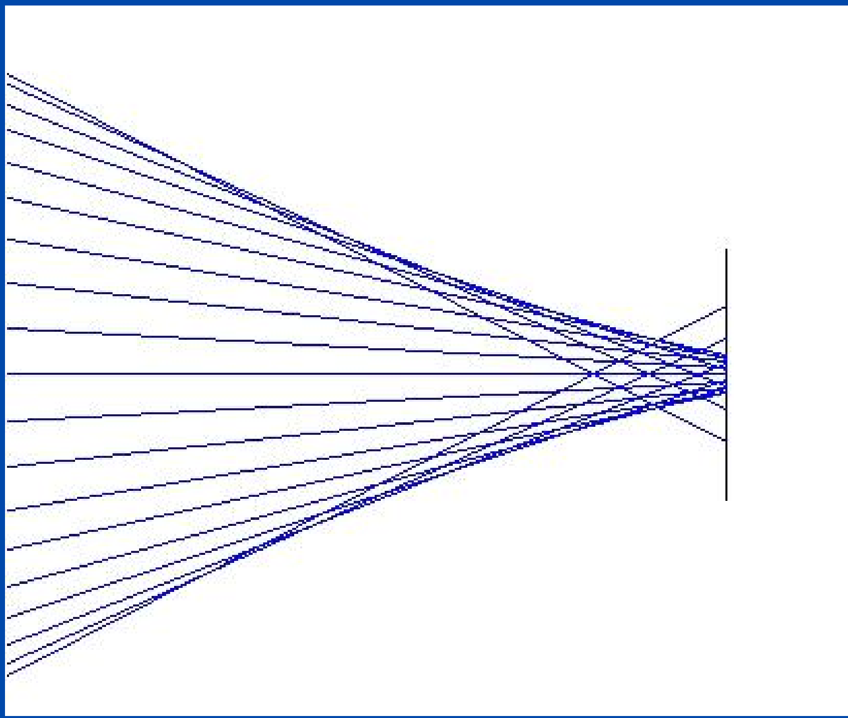


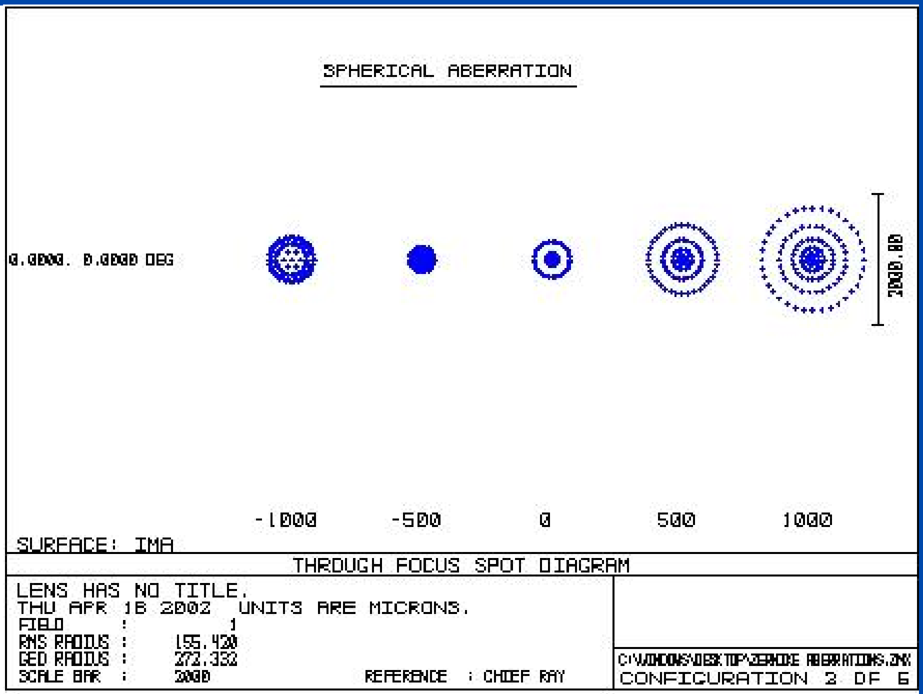
"Spot diagram": Image at different focus positions



Shows "spots" where rays would strike hypothetical detector

Spherical aberration

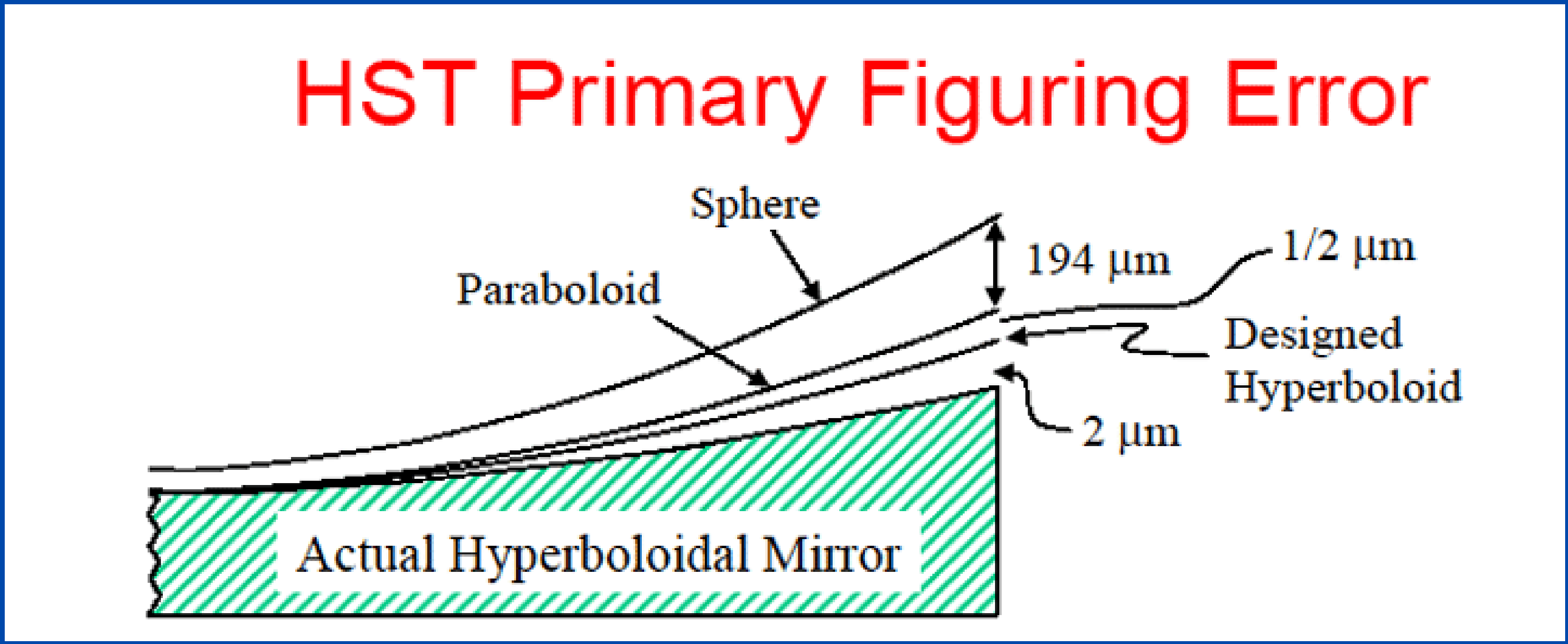




Rays from a spherically aberrated wavefront focus at different planes

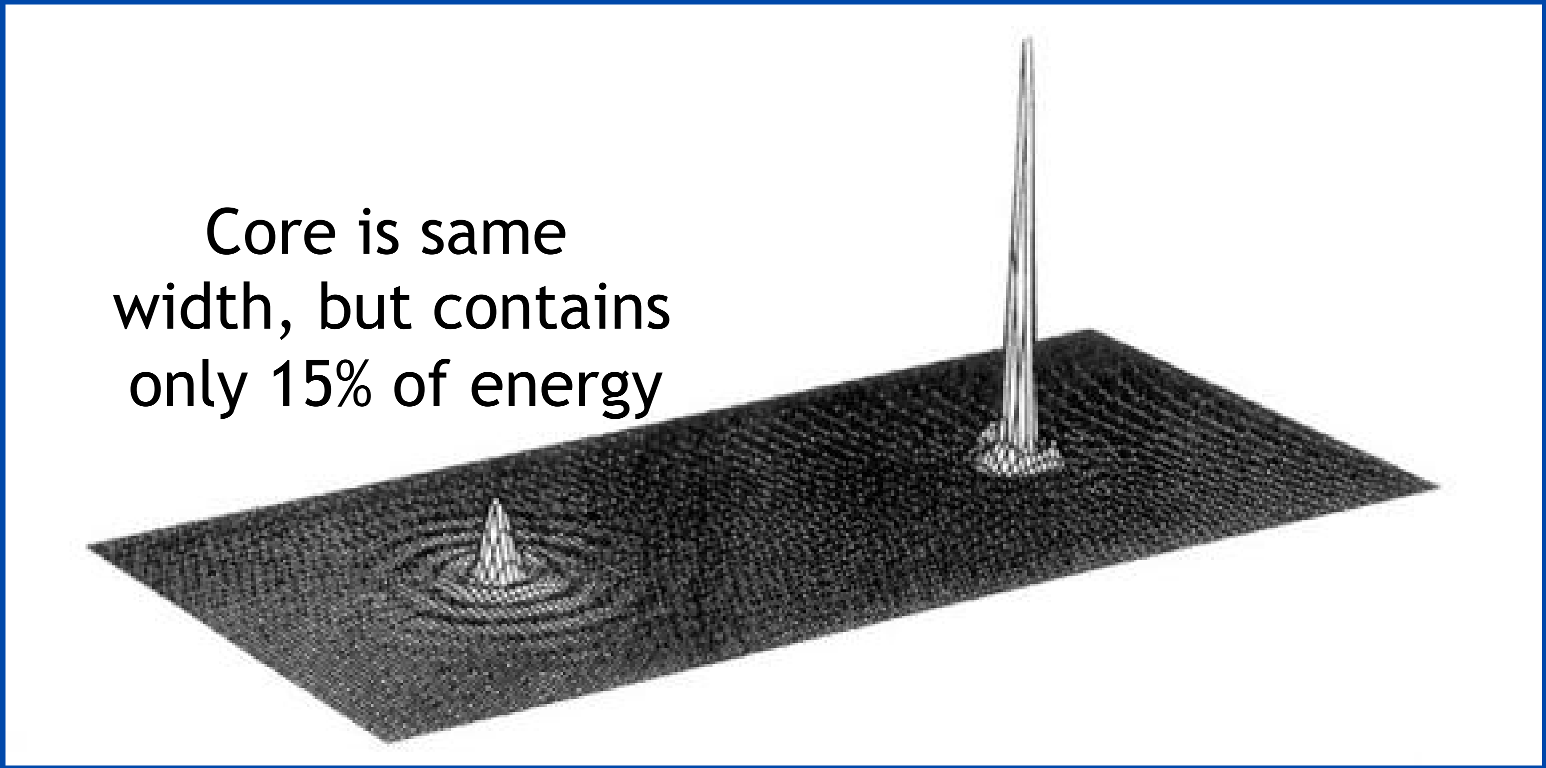
Through-focus spot diagram for spherical aberration

Hubble Space Telescope suffered from Spherical Aberration



- In a Cassegrain telescope, the hyperboloid of the primary mirror must match the specs of the secondary mirror. For HST they didn't match.

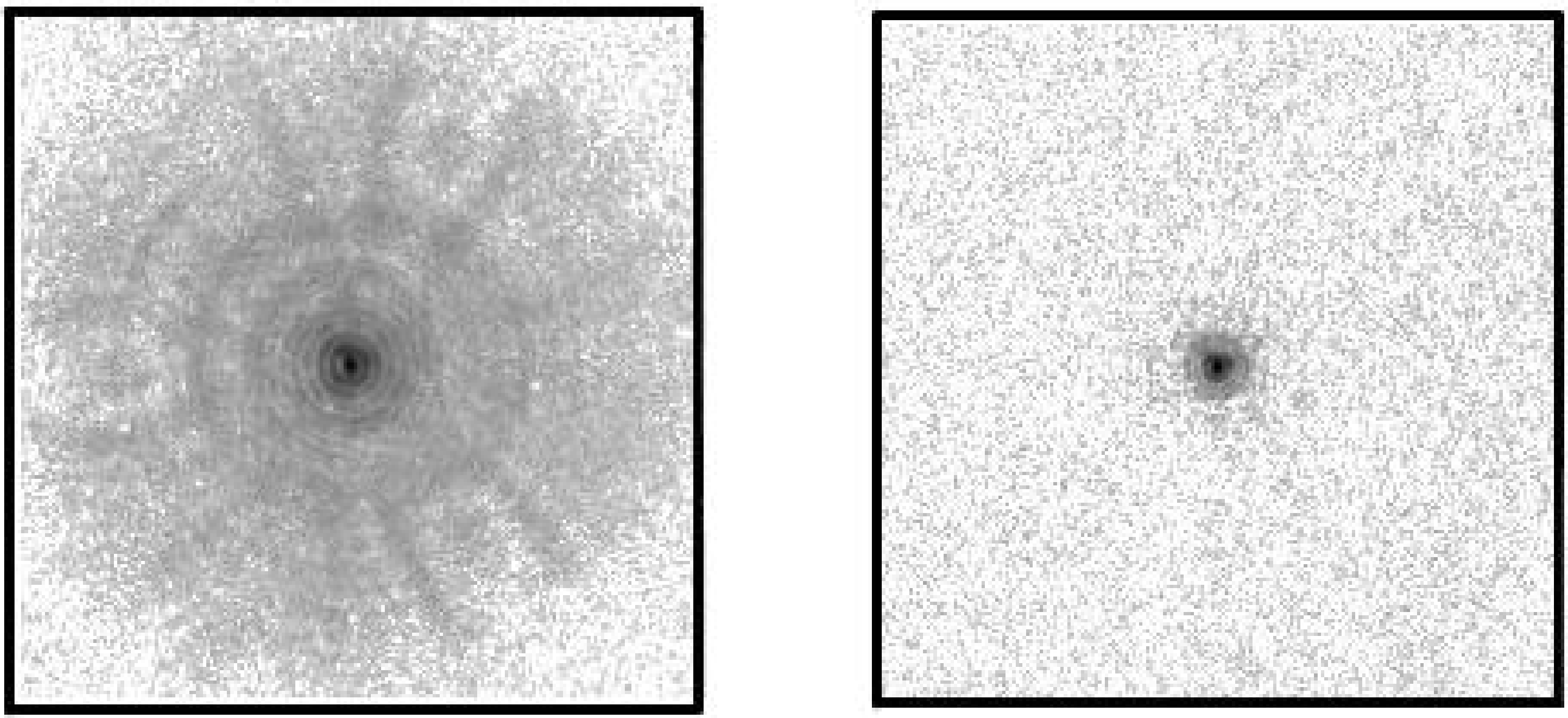
HST Point Spread Function (image of a point source)



Before COSTAR fix

After COSTAR fix

Point spread functions before and after spherical aberration was corrected

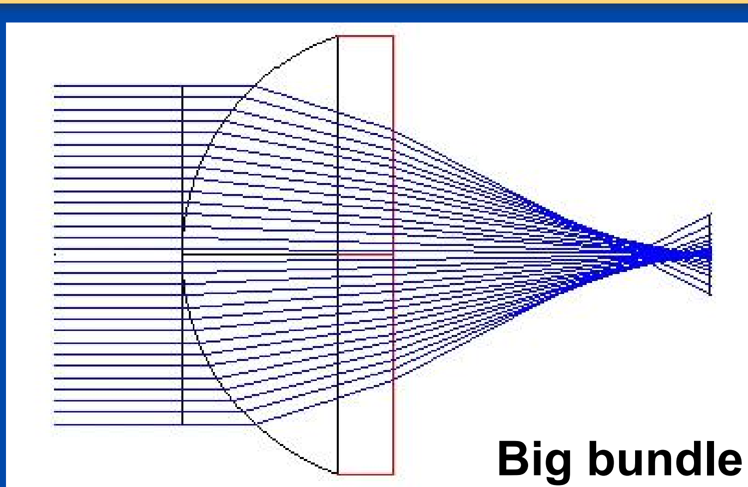


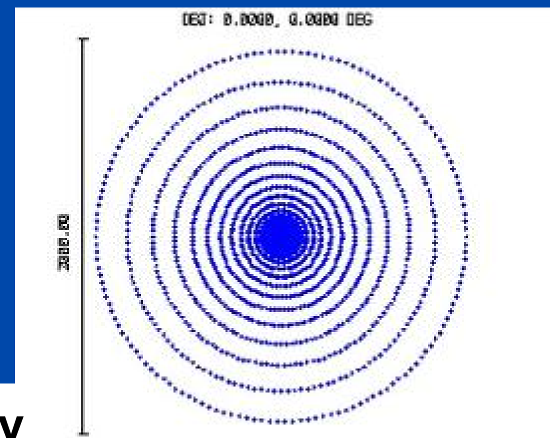
Central peak of uncorrected image (left) contains only 15% of central peak energy in corrected image (right)

Spherical aberration as "the mother of all other aberrations"

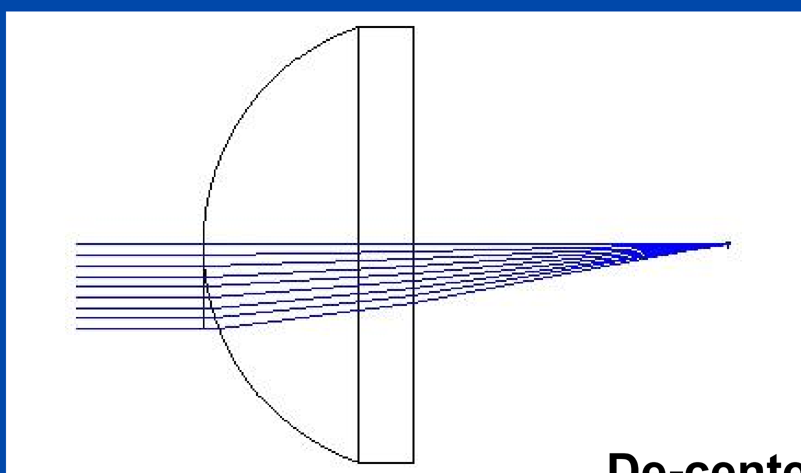
- Coma and astigmatism can be thought of as the aberrations from a de-centered bundle of spherically aberrated rays
- Ray bundle on axis shows spherical aberration only
- Ray bundle slightly de-centered shows coma
- Ray bundle more de-centered shows astigmatism
- All generated from subsets of a larger centered bundle of spherically aberrated rays
  - (diagrams follow)

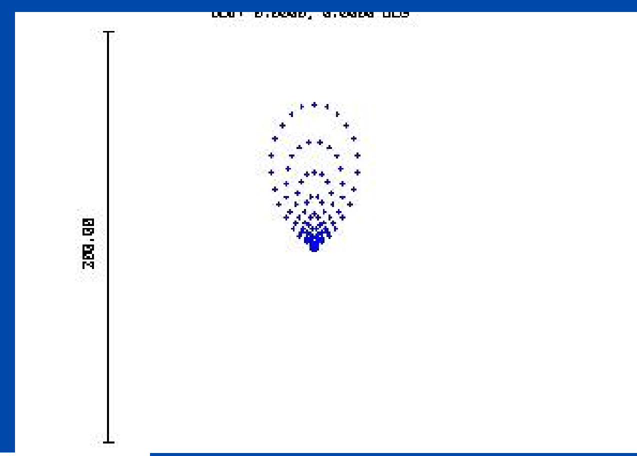
Spherical aberration as the mother of coma





Big bundle of spherically aberrated rays

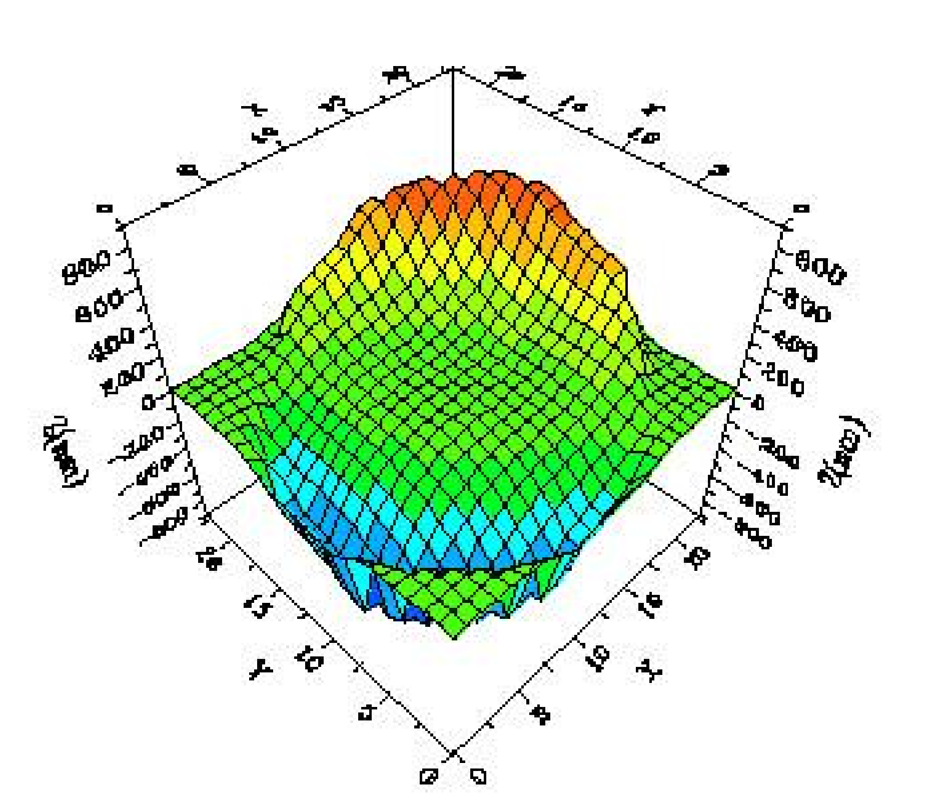




De-centered subset of rays produces coma

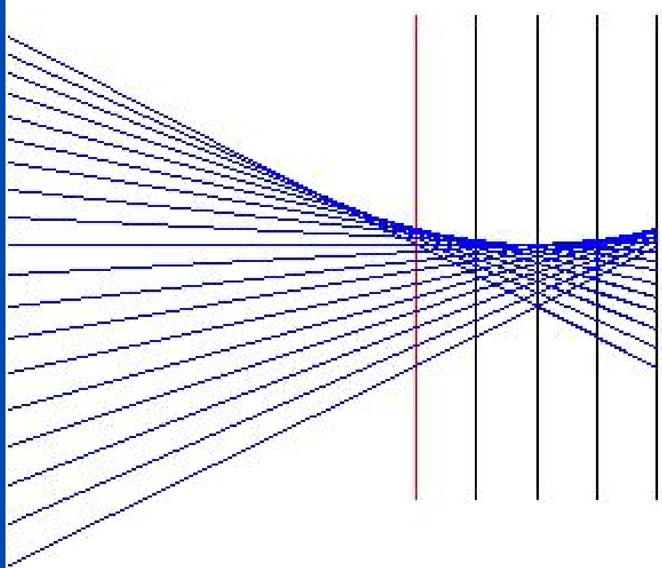
Coma

- "Comet"-shaped spot
- Chief ray is at apex of coma pattern
- Centroid is shifted from chief ray!
- Centroid shifts with change in focus!

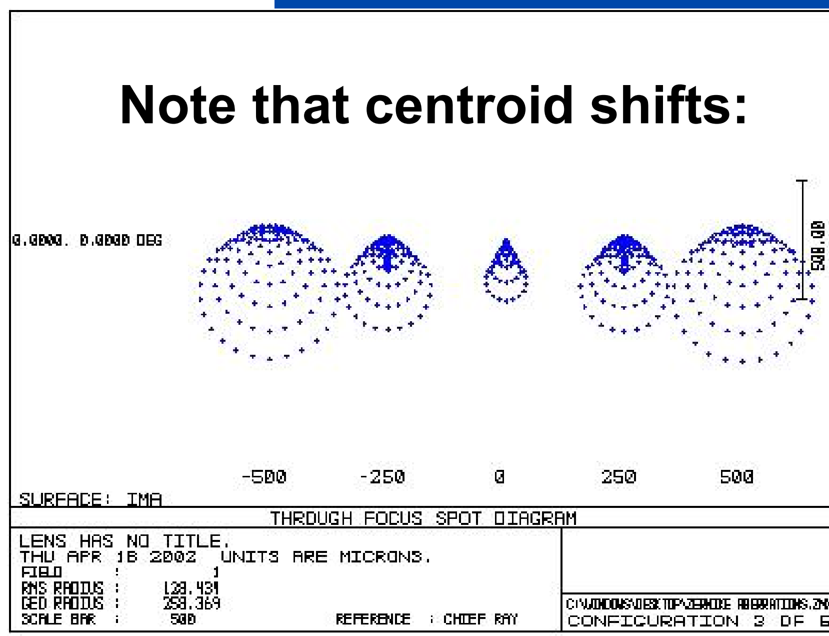


Wavefront

Coma

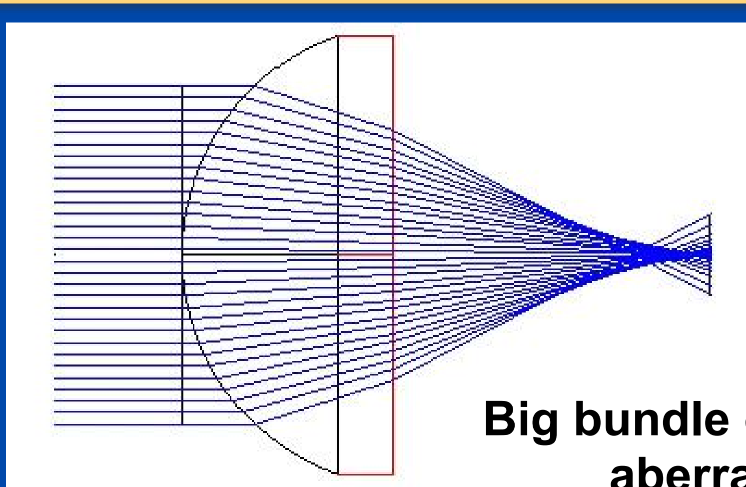


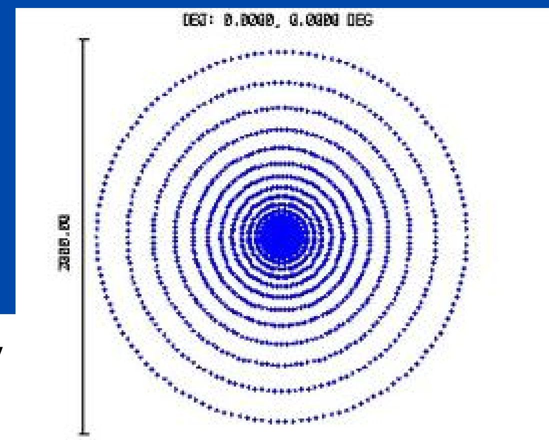
Rays from a comatic wavefront



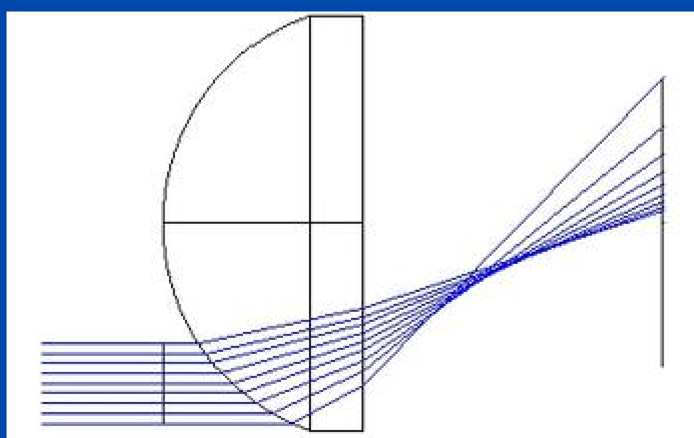
Through-focus spot diagram for coma

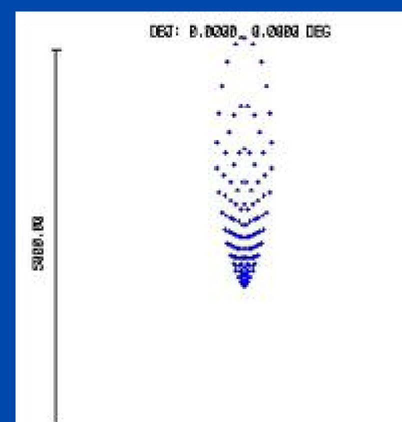
Spherical aberration as the mother of astigmatism





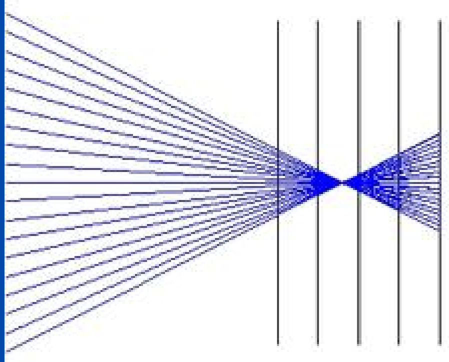
Big bundle of spherically aberrated rays



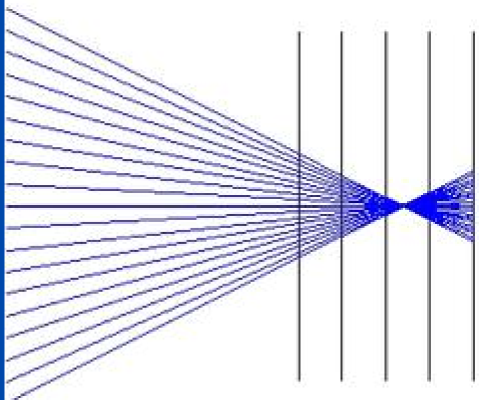


More-decentered subset of rays produces astigmatism

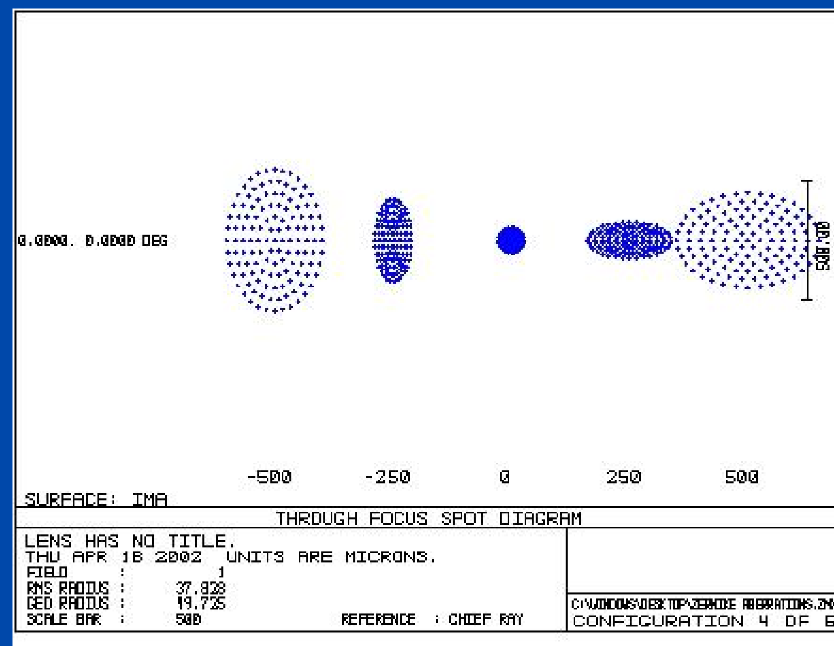
Astigmatism



Top view of rays



Side view of rays



Through-focus spot diagram for astigmatism

Different view of astigmatism

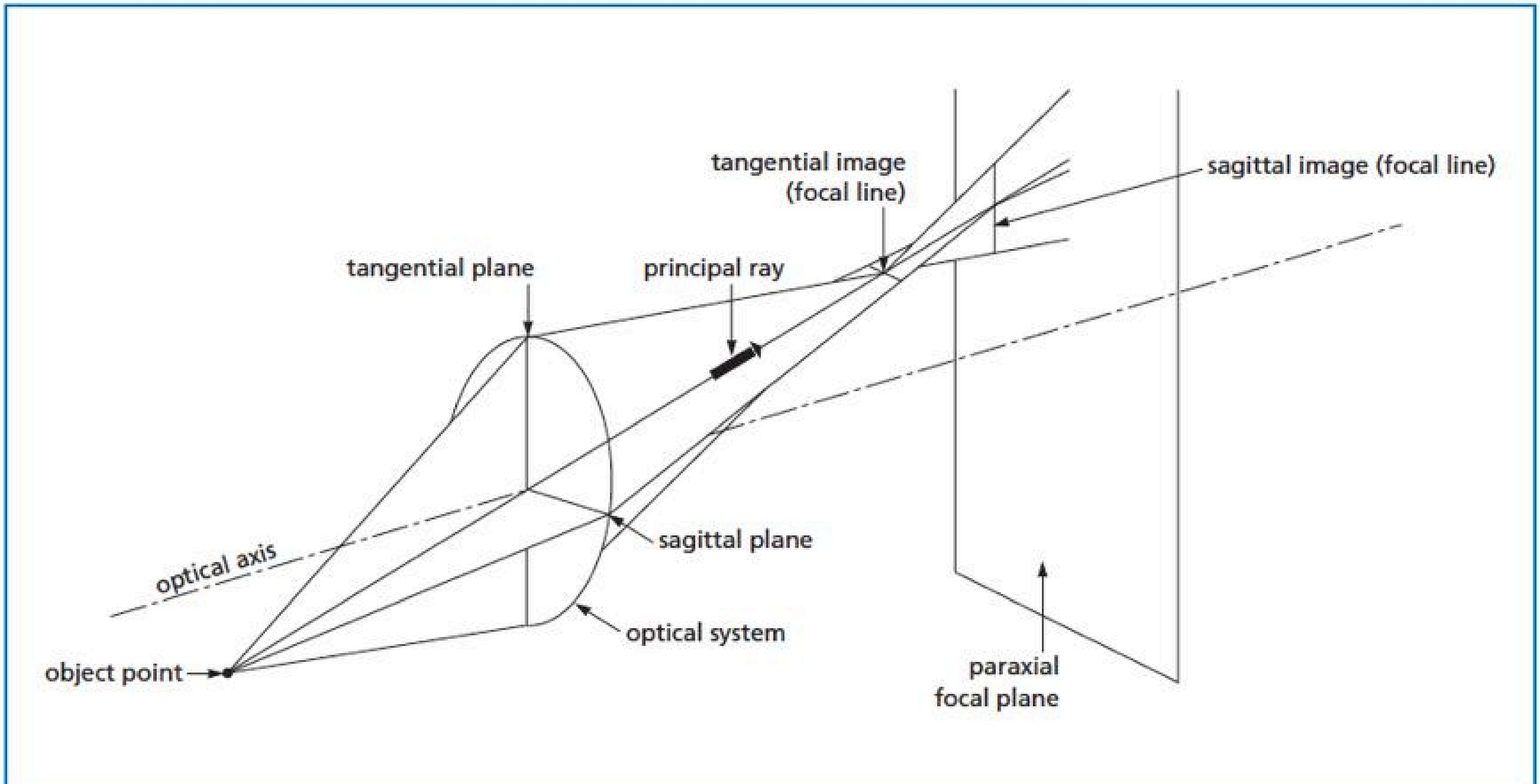
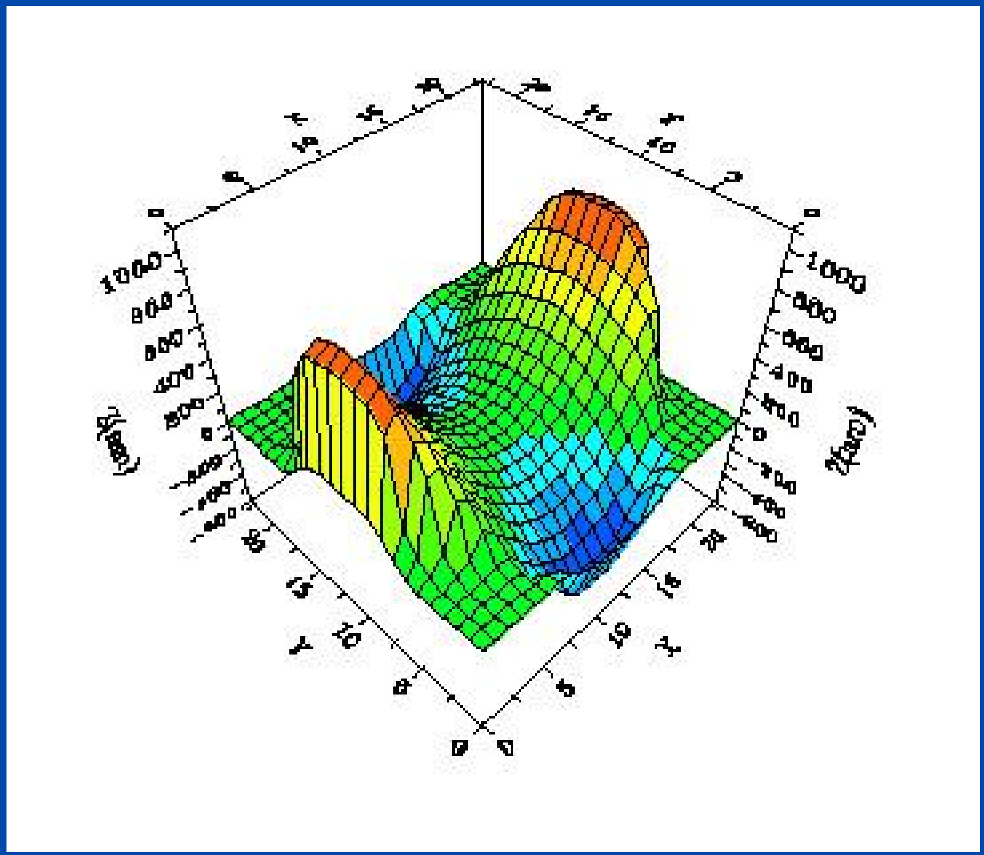


Figure 1.16 **Astigmatism represented by sectional views**

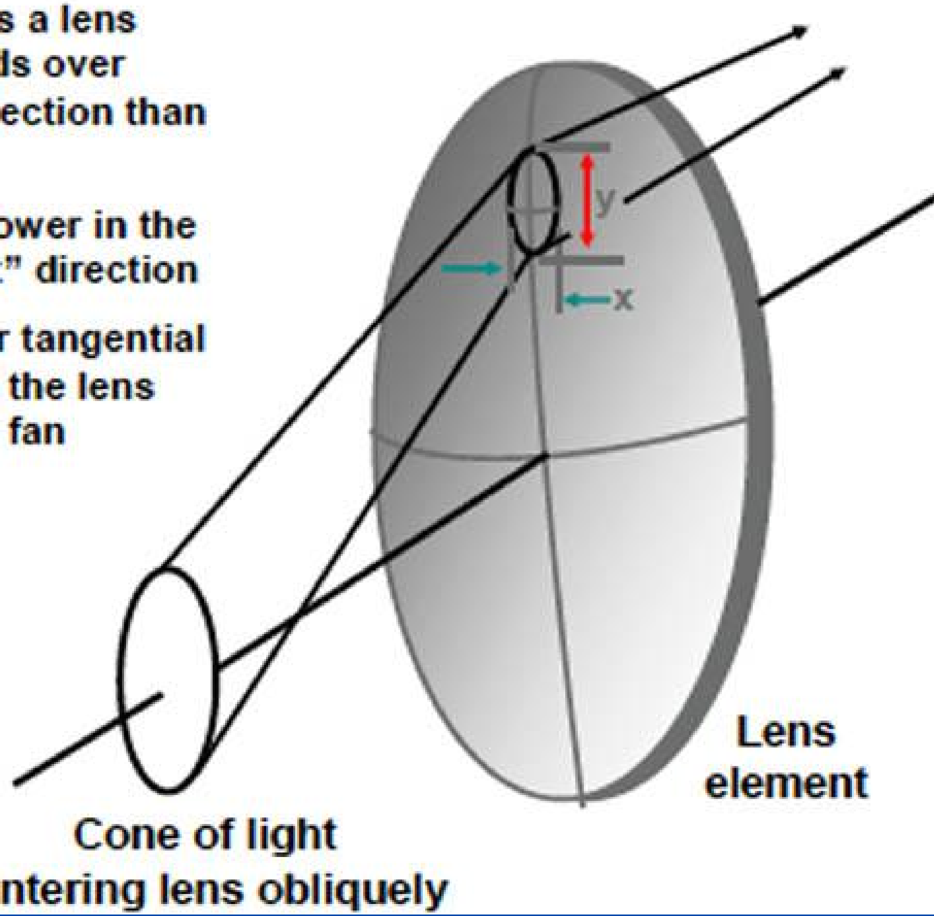
Credit: Melles-Griot

Wavefront for astigmatism



Where does astigmatism come from?

- When a cone of light enters a lens surface obliquely, it extends over more surface in the "y" direction than I the "x" direction
- This will introduce more power in the "y" direction than in the "x" direction
- The result is that the "y" or tangential ray fan will focus closer to the lens than the "x" or sagittal ray fan
- This is astigmatism



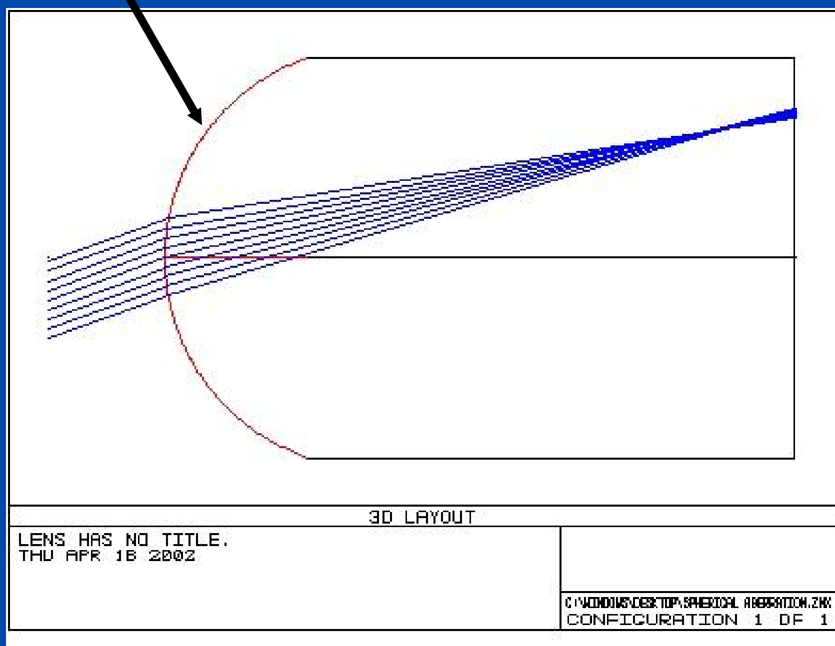
From Ian McLean, UCLA

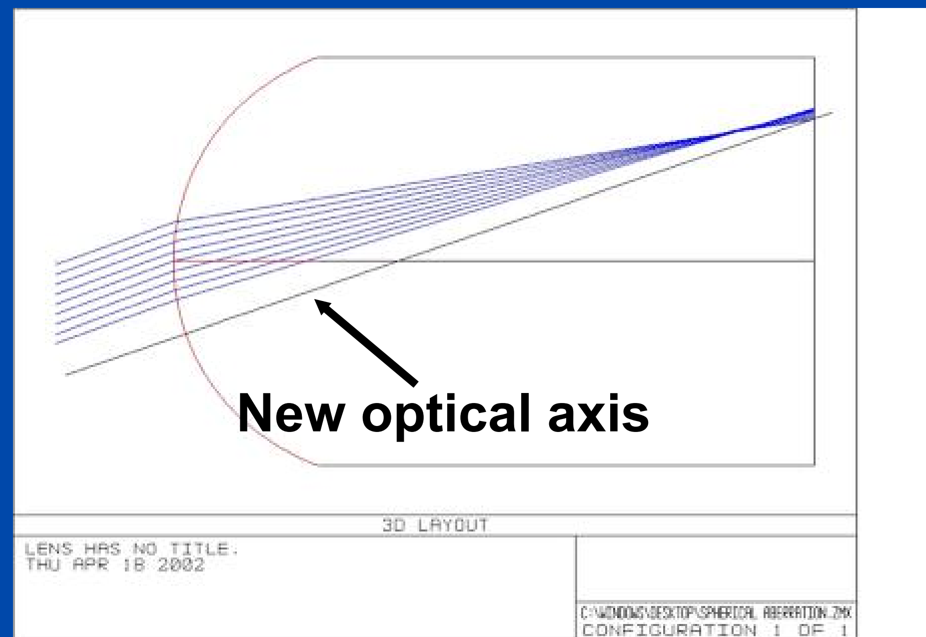
Concept Question

- How do you suppose eyeglasses correct for astigmatism?

Off-axis object is equivalent to having a de-centered ray bundle

Spherical surface





Ray bundle from an off-axis object. How to view this as a de-centered ray bundle?

For any field angle there will be an optical axis, which is ⊥ to the surface of the optic and // to the incoming ray bundle. The bundle is de-centered wrt this axis.

Zernike Polynomials

- Convenient basis set for expressing wavefront aberrations over a *circular pupil*
- Zernike polynomials are orthogonal to each other
- A few different ways to normalize – always check definitions!

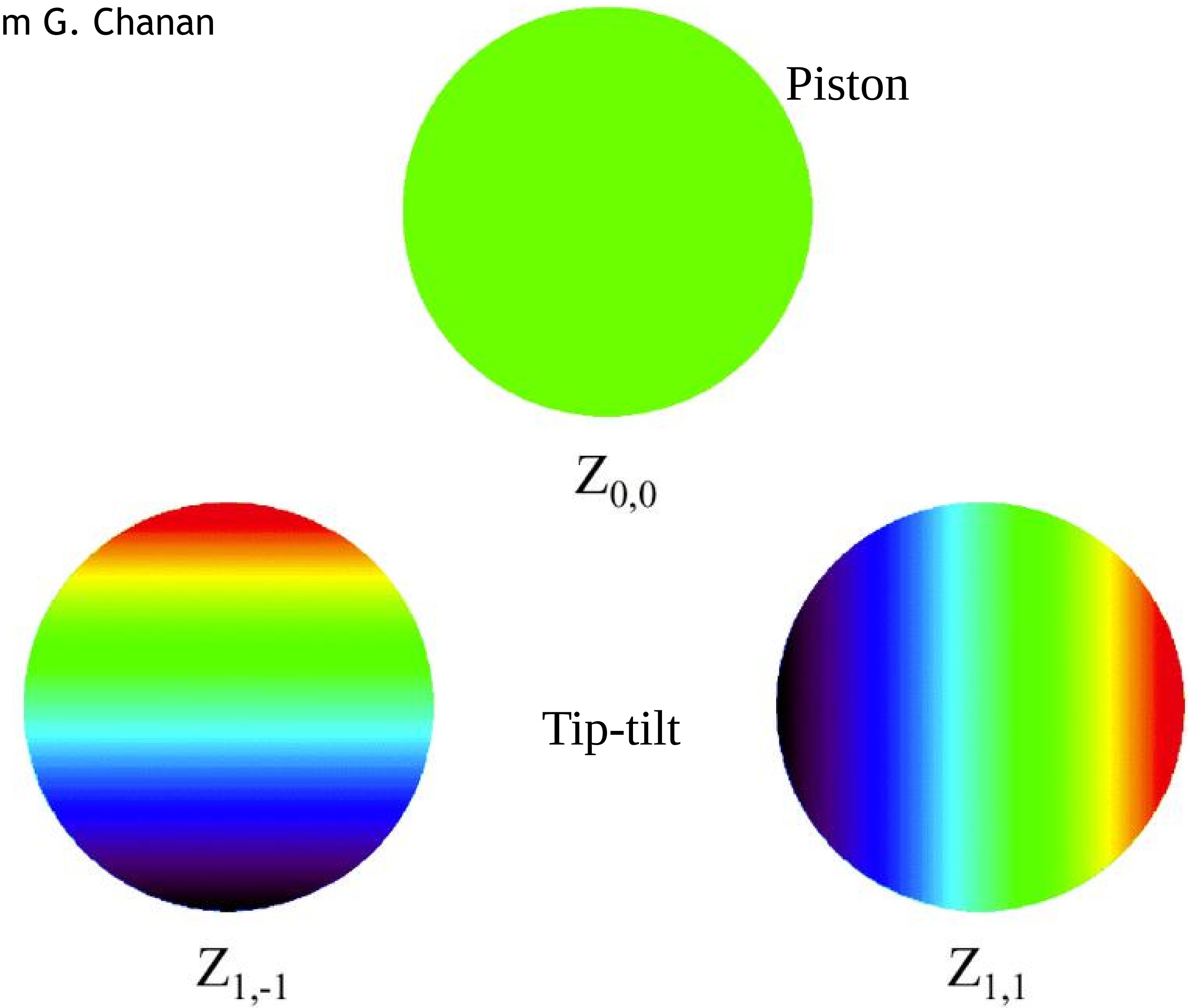
Expansion of the Phase in Zernike Polynomials

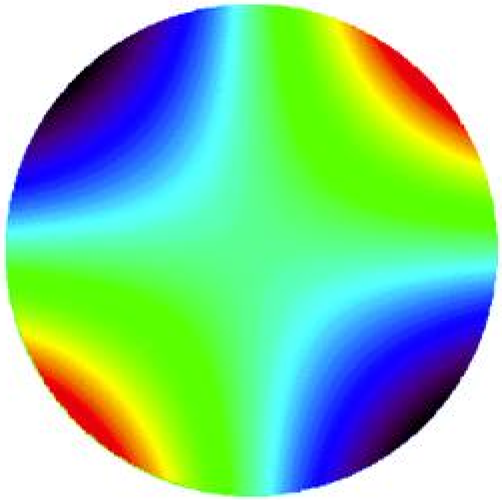
An alternative characterization of the phase comes from expanding φ in terms of a complete set of functions and then characterizing the coefficients of the expansion:

$$\varphi\,(r,\theta) = \Sigma\, a_{m,n}\, Z_{m,n}(r,\theta)$$

$$
\begin{array}{lll}
Z_{0,0} = 1 & & \text{piston} \\
\left.\begin{array}{l} Z_{1,-1} = 2\, r \sin\theta \\ Z_{1,1} = 2\, r \cos\theta \end{array}\right\} & & \text{tip/tilt} \\
Z_{2,-2} = \sqrt{6}\, r^2 \sin 2\theta & & \text{astigmatism} \\
Z_{2,0} = \sqrt{3}\,(2r^2 - 1) & & \text{focus} \\
Z_{2,2} = \sqrt{6}\, r^2 \cos 2\theta & & \text{astigmatism}
\end{array}
$$

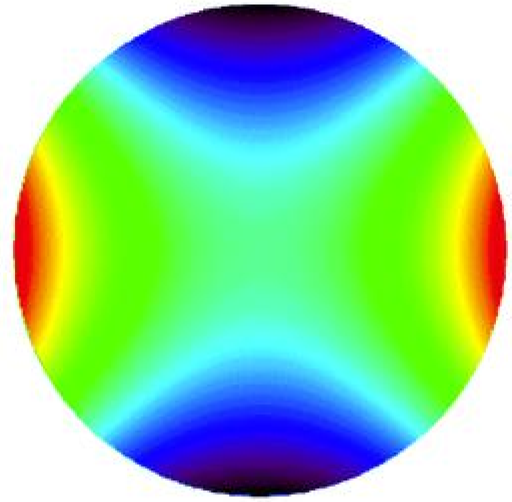
From G. Chanan



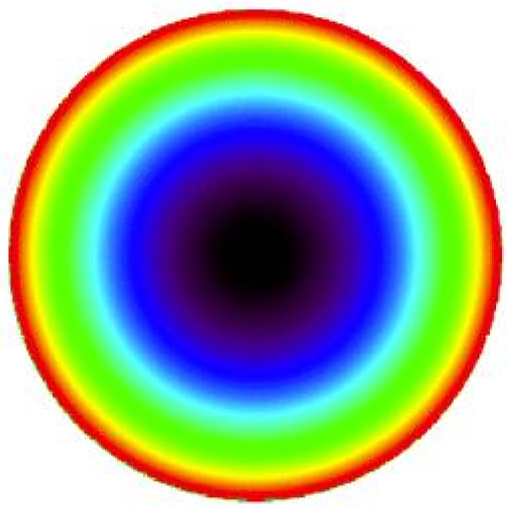


$Z_{2,-2}$

Astigmatism
(3rd order)

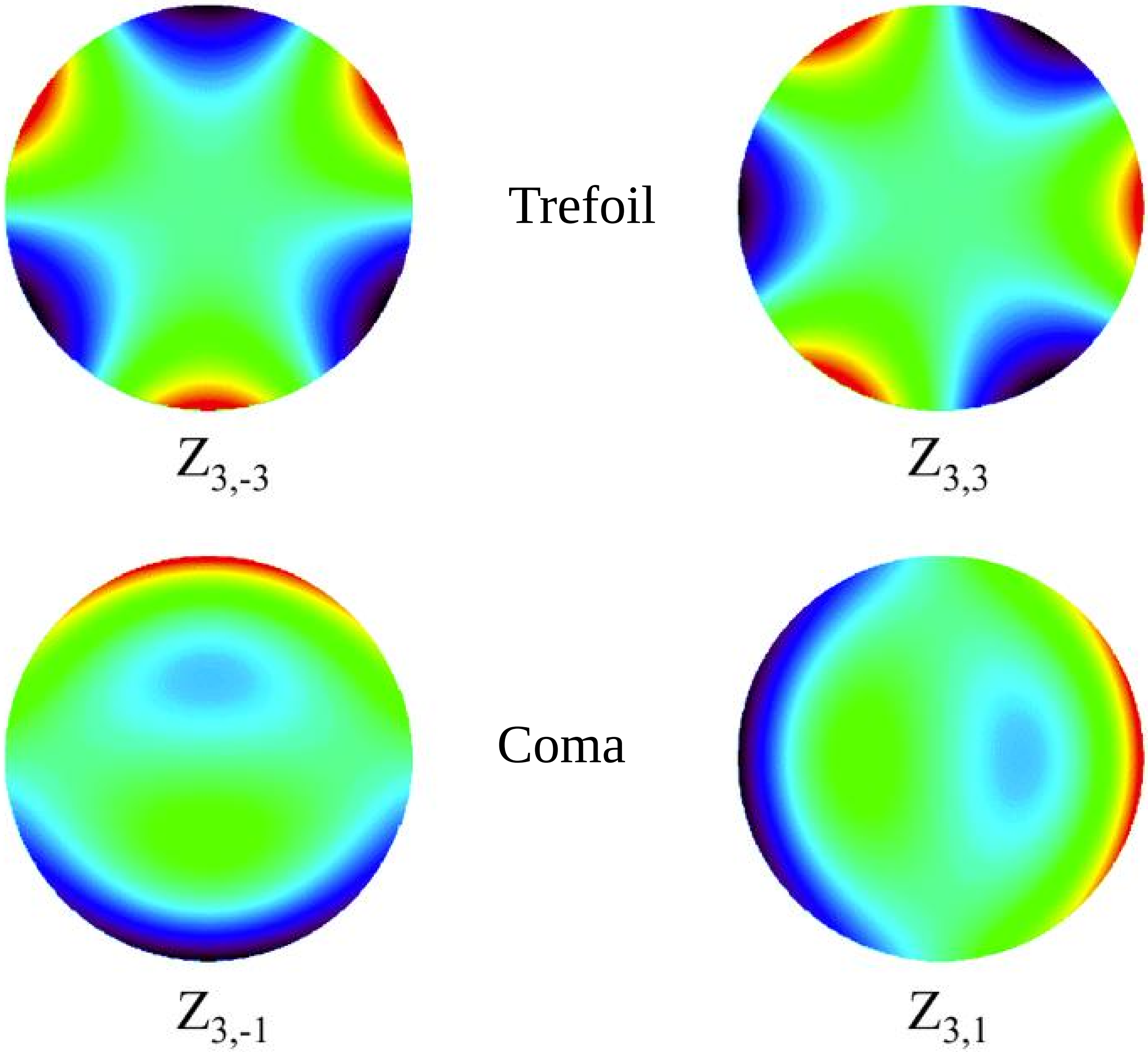


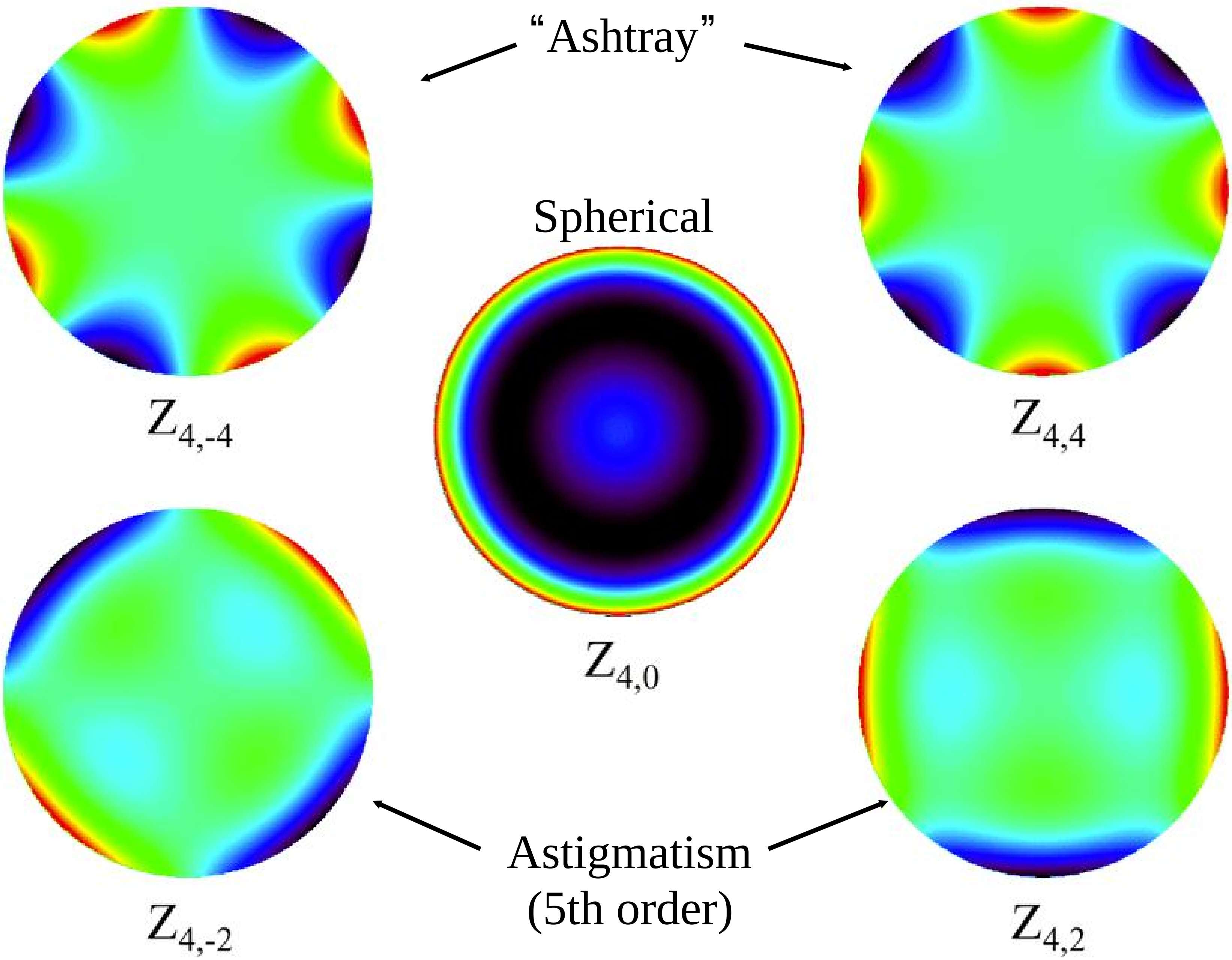
$Z_{2,2}$

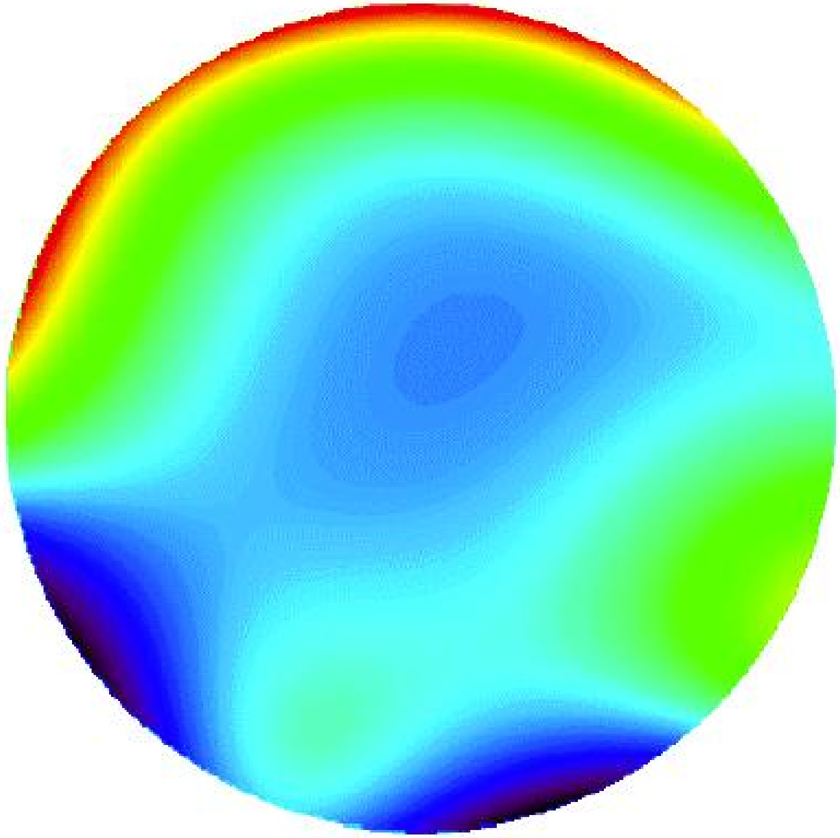


$Z_{2,0}$

Defocus

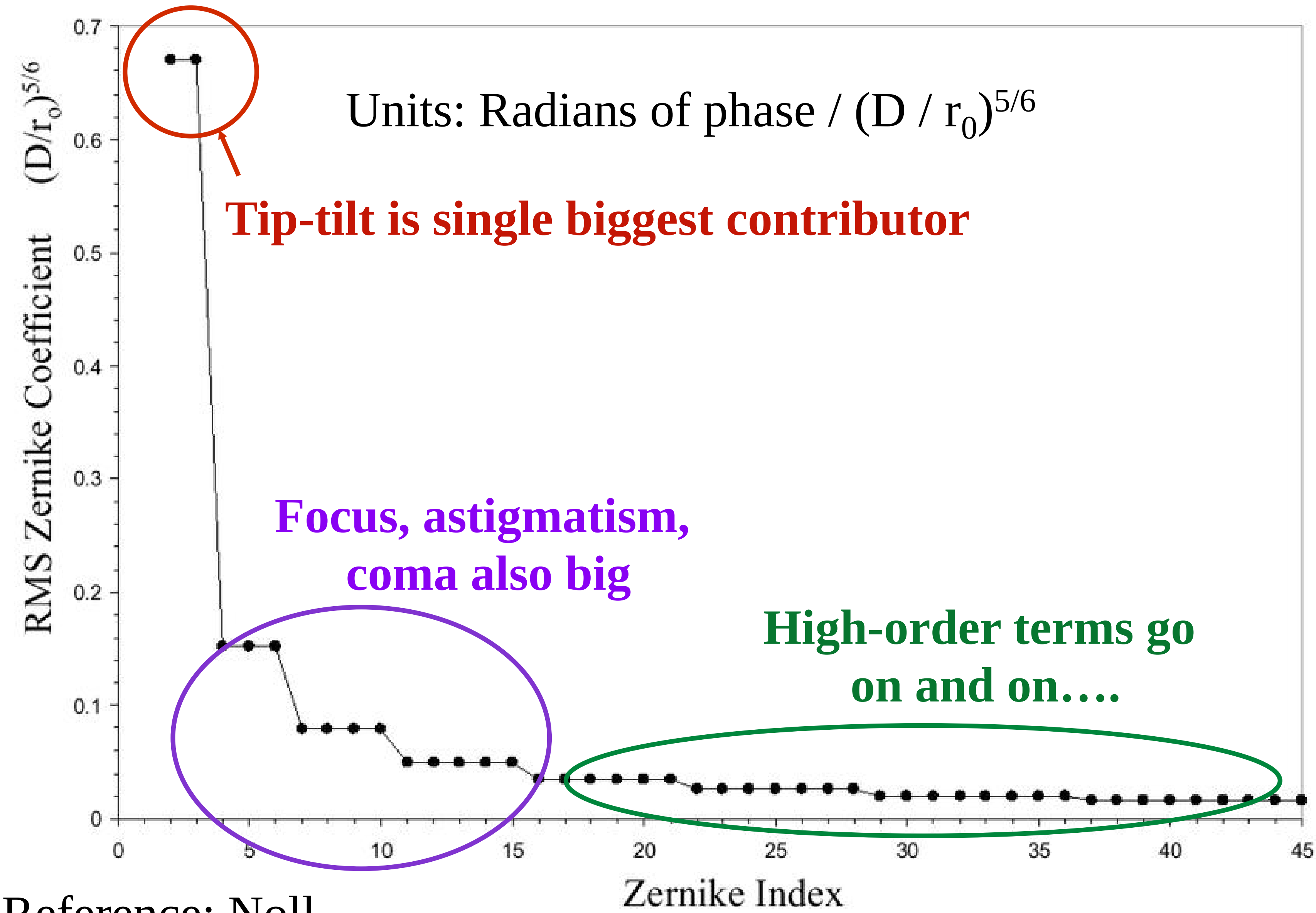

Trefoil
$Z_{3,-3}$
$Z_{3,3}$
Coma
$Z_{3,-1}$
$Z_{3,1}$


"Ashtray"
Spherical
$Z_{4,-4}$
$Z_{4,4}$
$Z_{4,0}$
$Z_{4,-2}$
$Z_{4,2}$
Astigmatism
(5th order)



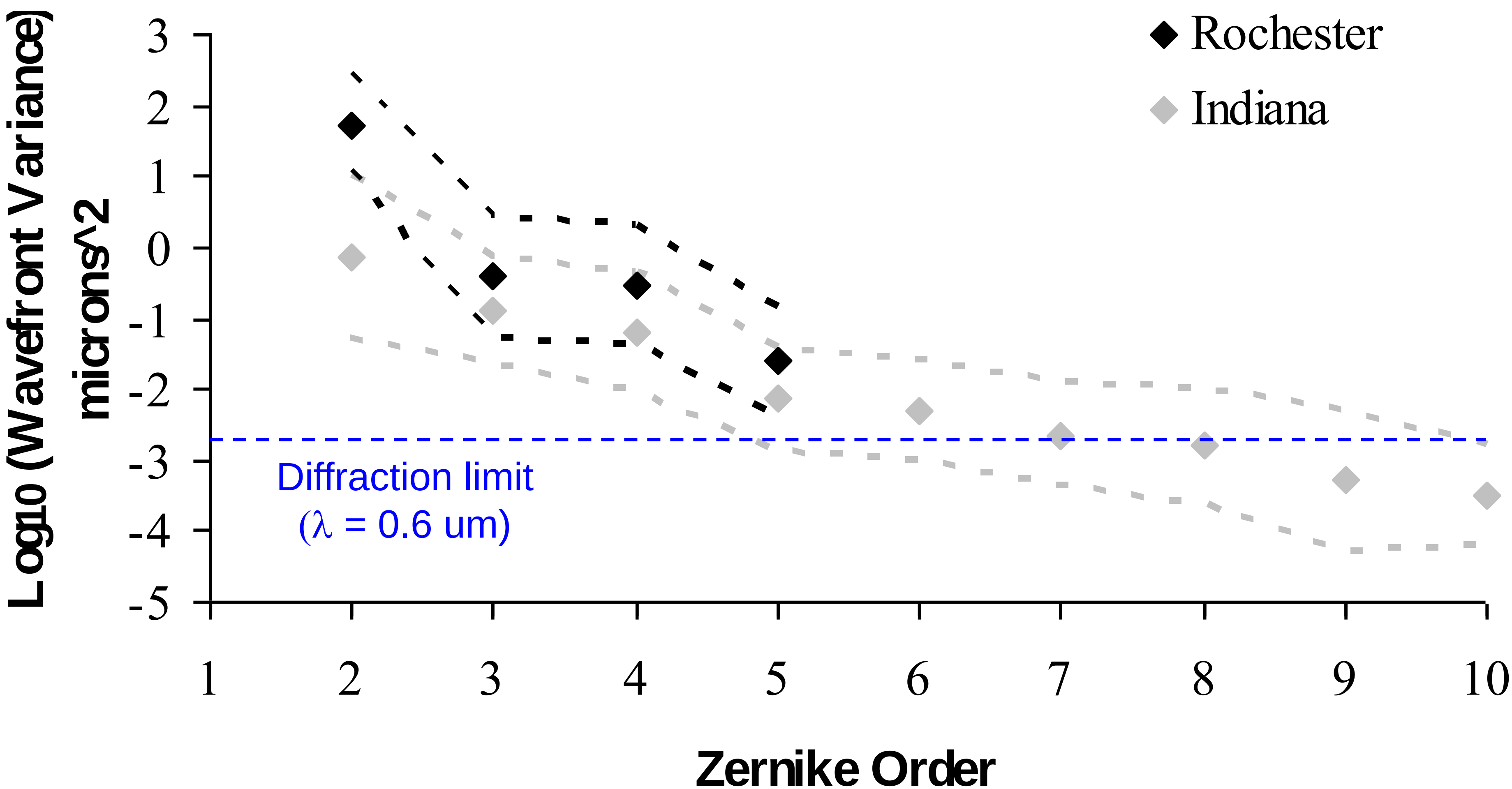
Random Zernikes

Atmospheric Zernike Coefficients



Reference: Noll

Aberrations in two populations of 70 normal eyes for 7.5 mm pupil



N. Doble & D.T. Miller, “Vision correctors for vision science,“ Chapter 4, Adaptive Optics for Vision Science (2006).

N. Doble, *et al.*, "Requirements for discrete actuator and segmented wavefront correctors for aberration compensation in two large populations of human eyes," *Appl. Opt.* 46, 4501-4514 (2007),

Seidel polynomials vs. Zernike polynomials

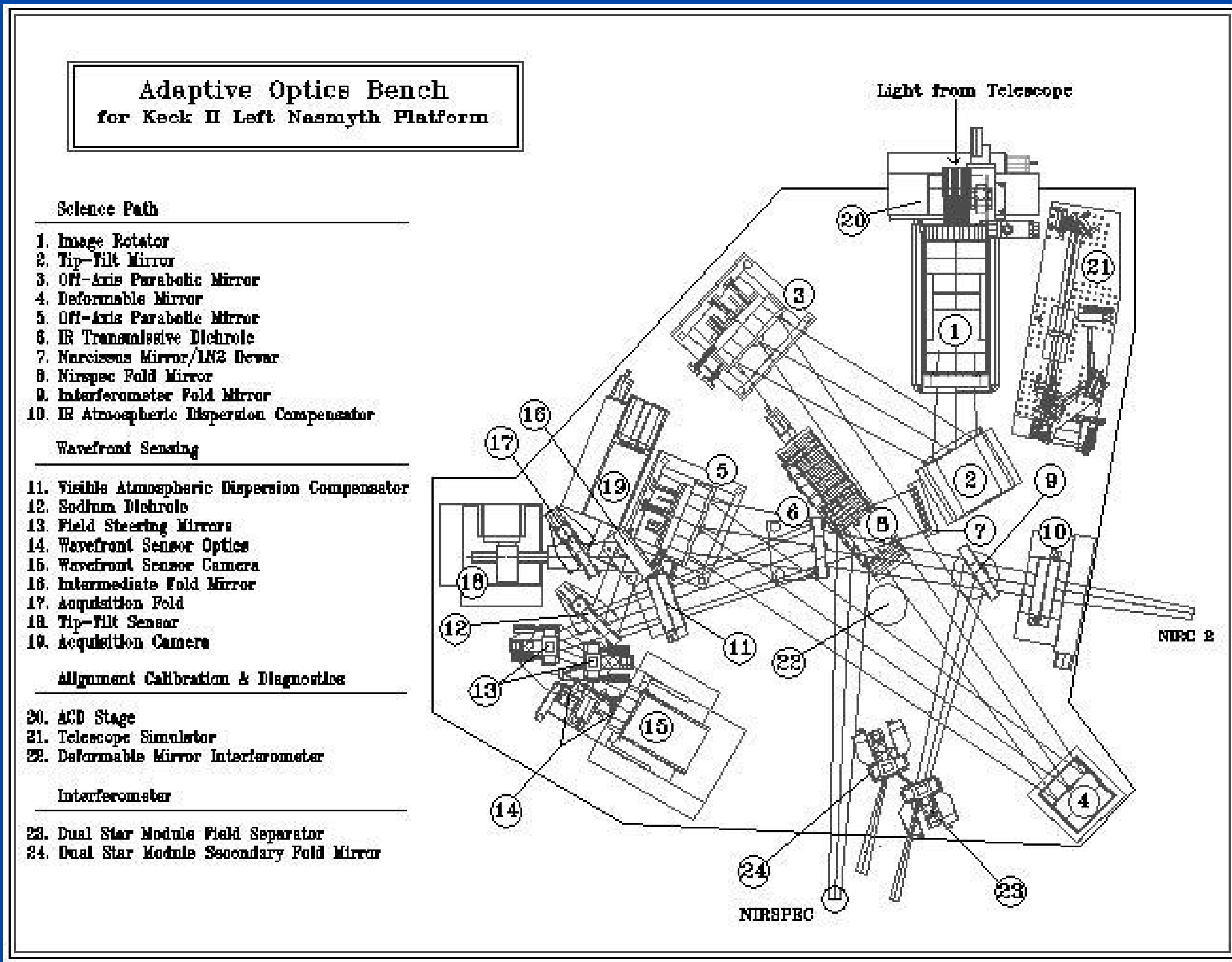
- Seidel polynomials also describe aberrations
- At first glance, Seidel and Zernike aberrations look very similar
- Zernike aberrations are an orthogonal set of functions used to decompose a given wavefront *at a given field point* into its components
  - Zernike modes add to the Seidel aberrations the correct amount of low-order modes to minimize rms wavefront error
- Seidel aberrations are used in optical design to predict the aberrations in a design and how they will vary *over the system's field of view*
- The Seidel aberrations have an analytic field-dependence that is proportional to some power of field angle

References for Zernike Polynomials

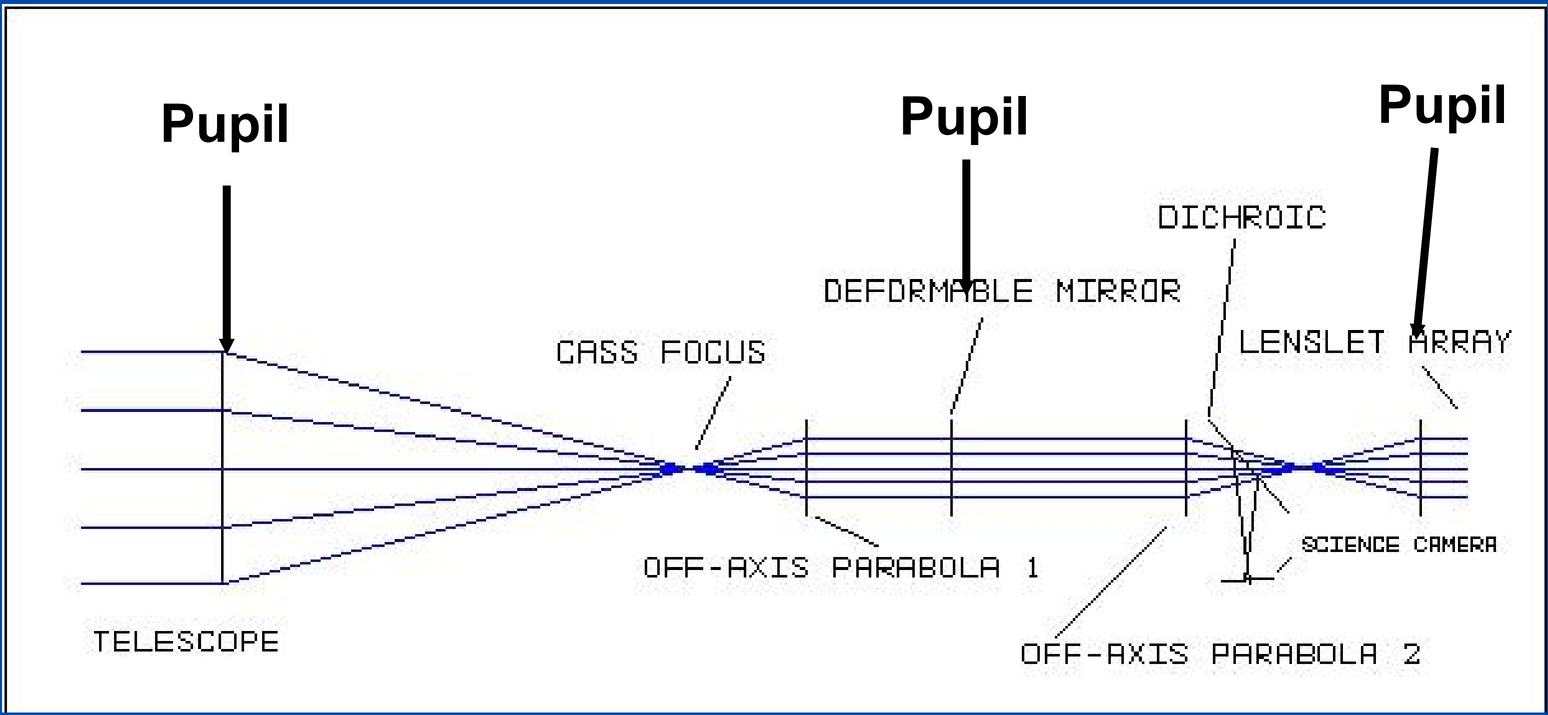
- Pivotal Paper: Noll, R. J. 1976, "Zernike polynomials and atmospheric turbulence", JOSA 66, page 207

- Books:
  - e.g. Hardy, Adaptive Optics, pages 95-96

Let's get back to design of AO systems
Why on earth does it look like this ??



Considerations in the optical design of AO systems: pupil relays

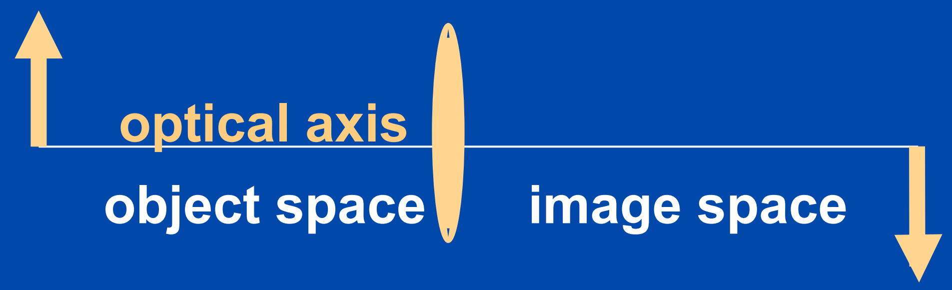


Deformable mirror and Shack-Hartmann lenslet array should be "optically conjugate to the telescope pupil."

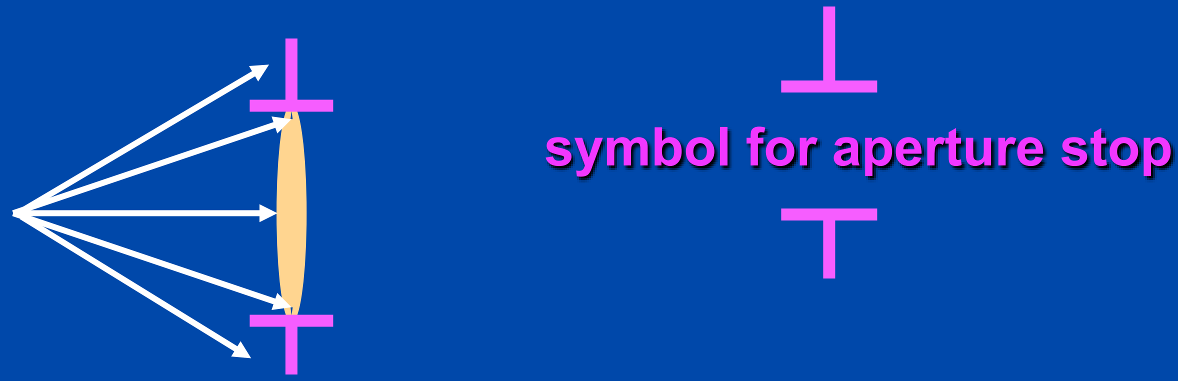
What does this mean?

Define some terms

- "Optically conjugate" = "image of...."



- "Aperture stop" = the aperture that limits the bundle of rays accepted by the optical system



- "Pupil" = image of aperture stop

So now we can translate:

- "The deformable mirror should be optically conjugate to the telescope pupil"

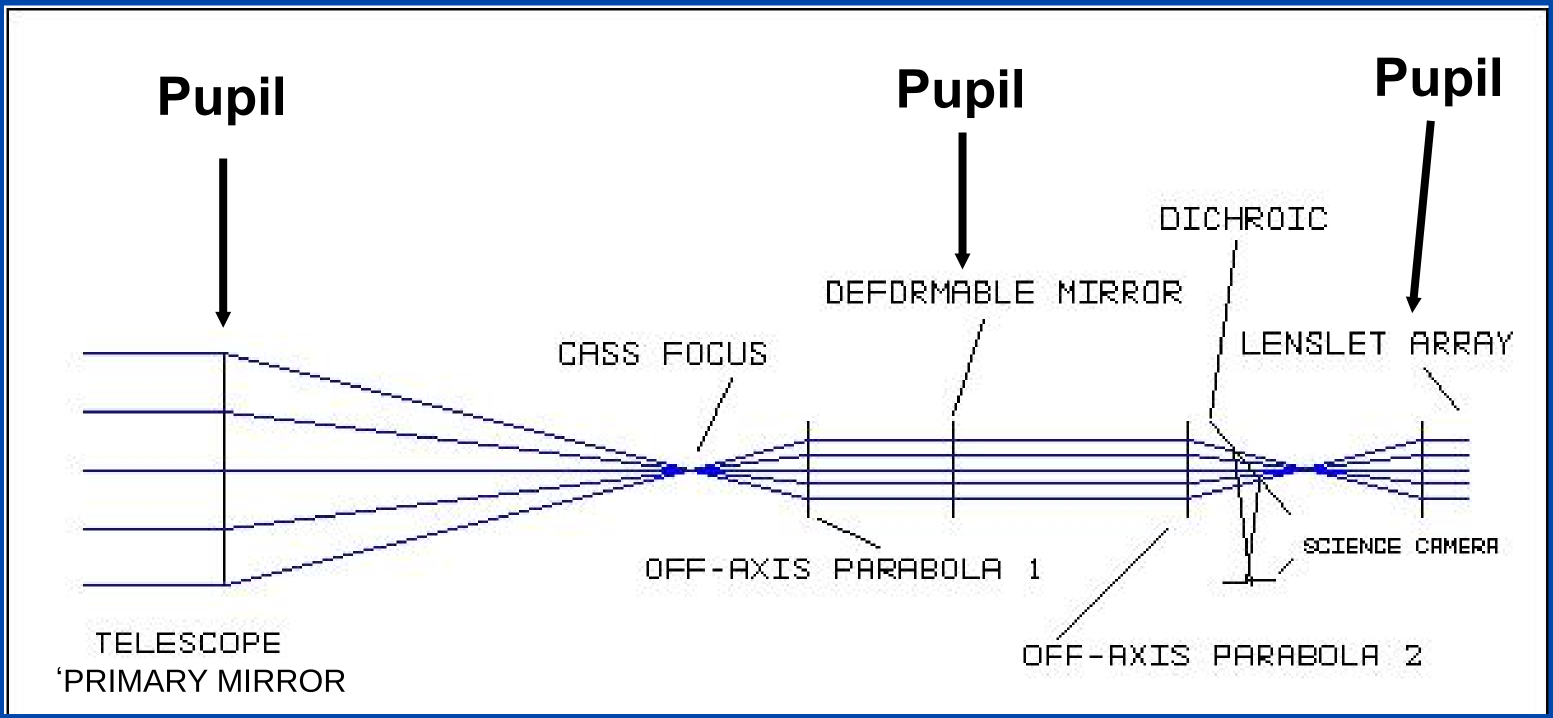
  means

- The surface of the deformable mirror is an image of the telescope pupil

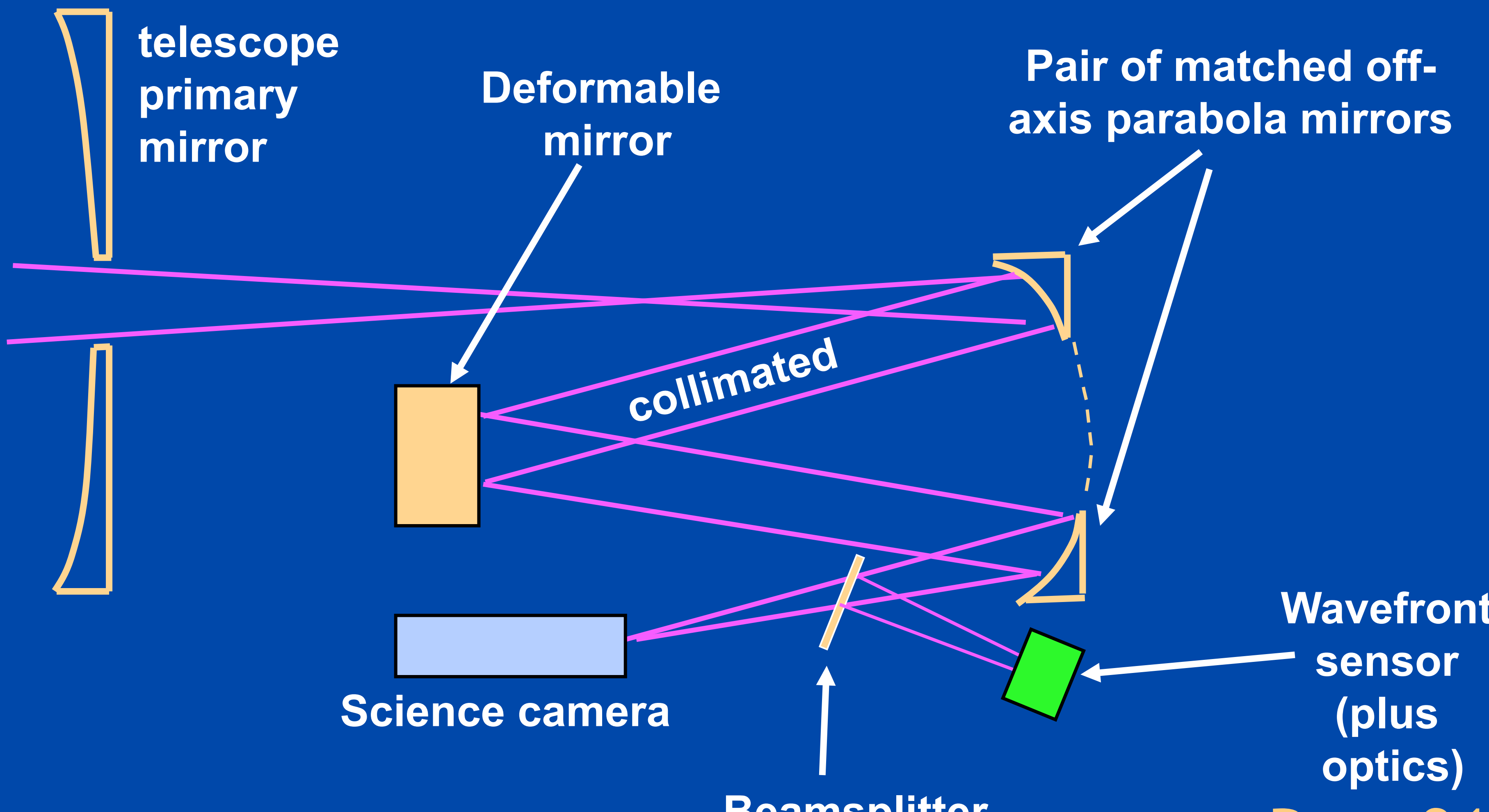
  where

- The pupil is an image of the aperture stop
  - **In practice, the pupil is usually the primary mirror of the telescope**

Considerations in the optical design of AO systems: "pupil relays"

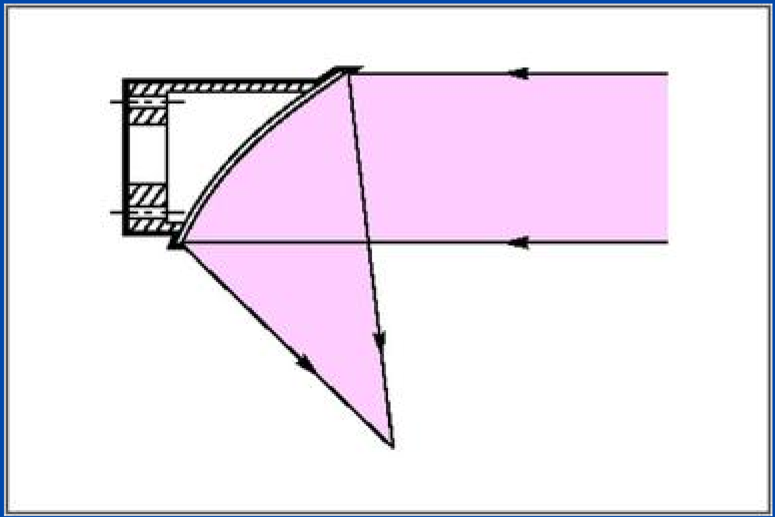


Typical optical design of AO system



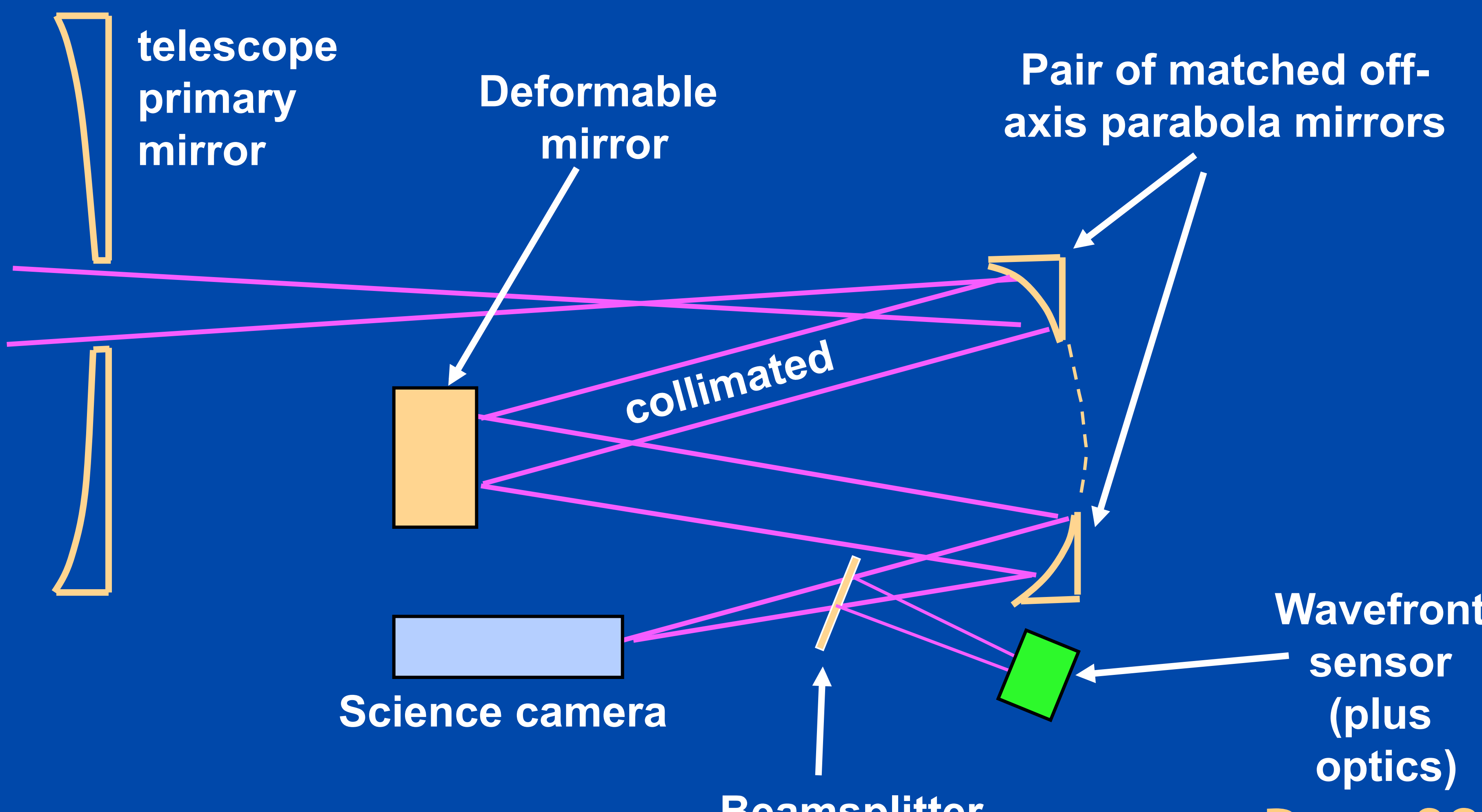
More about off-axis parabolas

- Circular cut-out of a parabola, off optical axis
- Frequently used in matched pairs (each cancels out the off-axis aberrations of the other) to first collimate light and then refocus it



SORL

Concept Question: what elementary optical calculations would you have to do, to lay out this AO system? (Assume you know telescope parameters, DM size)



Review of important points

- Both lenses and mirrors can focus and collimate light
- Equations for system focal lengths, magnifications are quite similar for lenses and for mirrors
- Telescopes are combinations of two or more optical elements
  - Main function: to gather lots of light
- Aberrations occur both due to your local instrument's optics and to the atmosphere
  - Can describe both with Zernike polynomials
- Location of pupils is important to AO system design